

# Statistical evaluation of NEMO OGCM outputs based on multi-order statistics

and some other works related to scales  
(downscaling and unresolved scales parameterization...)

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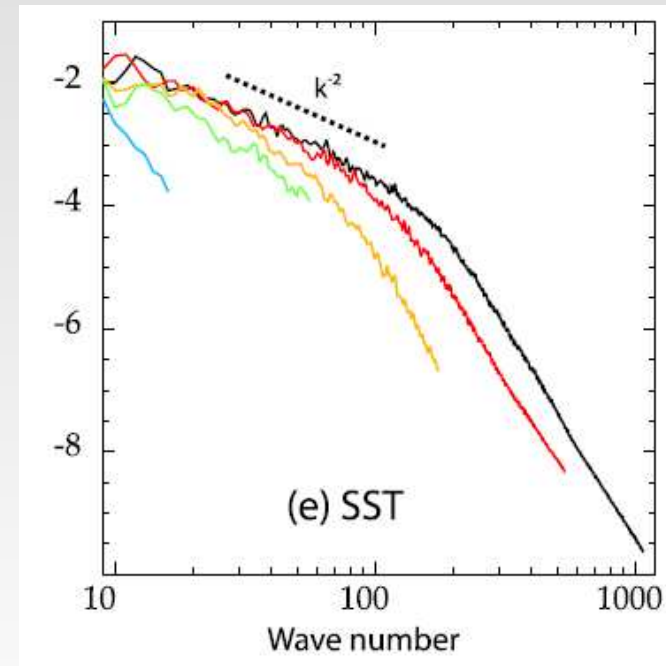
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# Topics

- I) Multi-scale, multi-order statistical evaluation of NEMO OGCM
  - Idealized configurations
  
- II) From multi-order statistics to multifractal downscaling
  - Method based on properties identified in I)
  - Applicability : more general than OGCMs
  
- III) Controlling unknown diffusion parameters in NEMO
  - A variational approach

# I) Statistical evaluation

- Geophysical flows are turbulent
- Scales related by remarkable symmetries
  - Energy/power spectra are scaling
- OGCM outputs should follow scaling statistics
  - Multi-scale evaluation tool
- Spectral tools are restrictive
  - Mono-order (quadratic)
  - Not the whole cdf



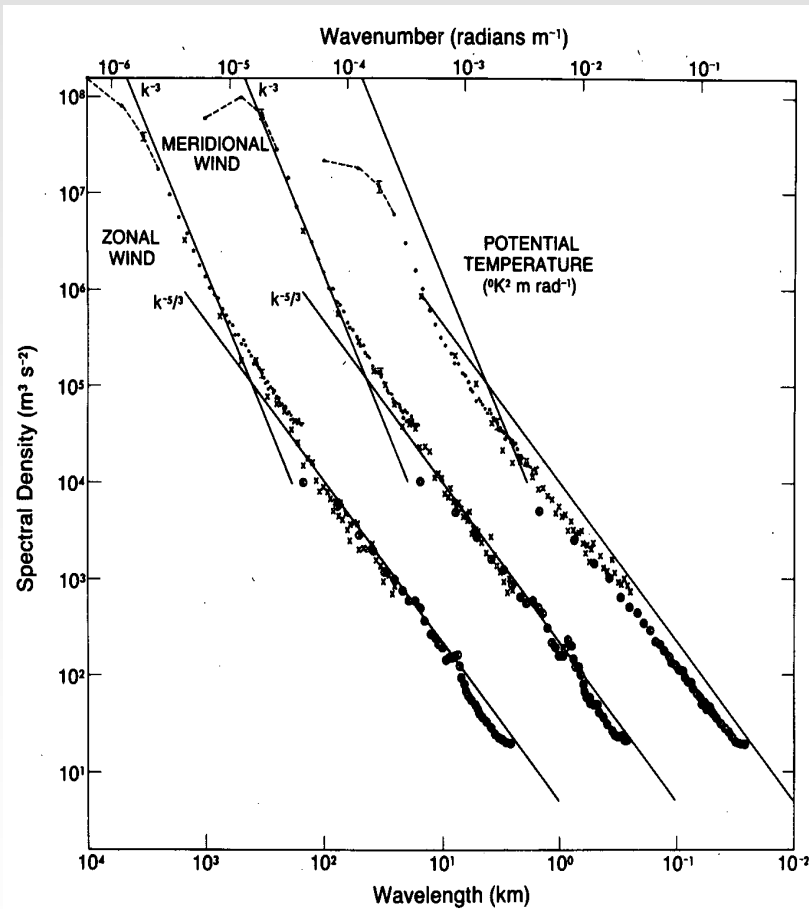
Example: NEMO-GYRE 1/54° spectrum (Lévy et al., 2012)

Suggestion: use multi-order statistics across different scales to generalize spectral analysis

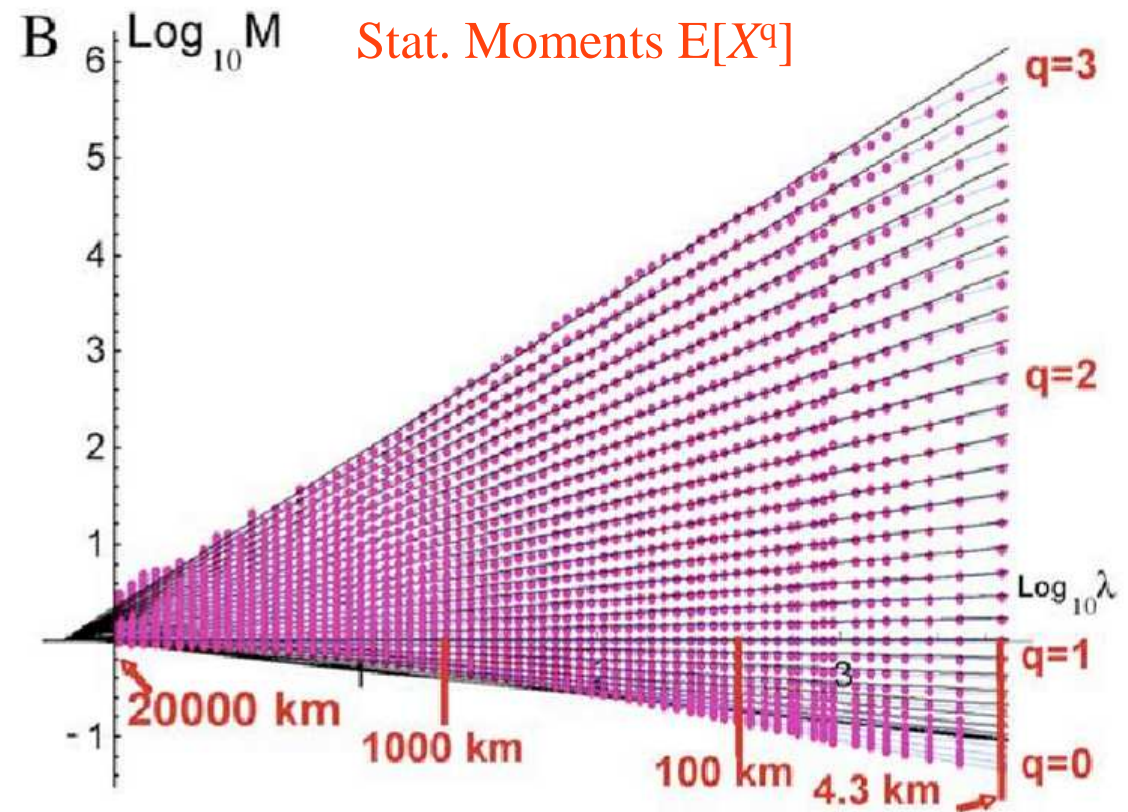
# From mono- to multi-order scaling : atmospheric data

Spectra

« Multifractals » (Schertzer & Lovejoy, 1987)



Spectra GASP (Gage & Nastrom, 1985; Lilly, 1989)



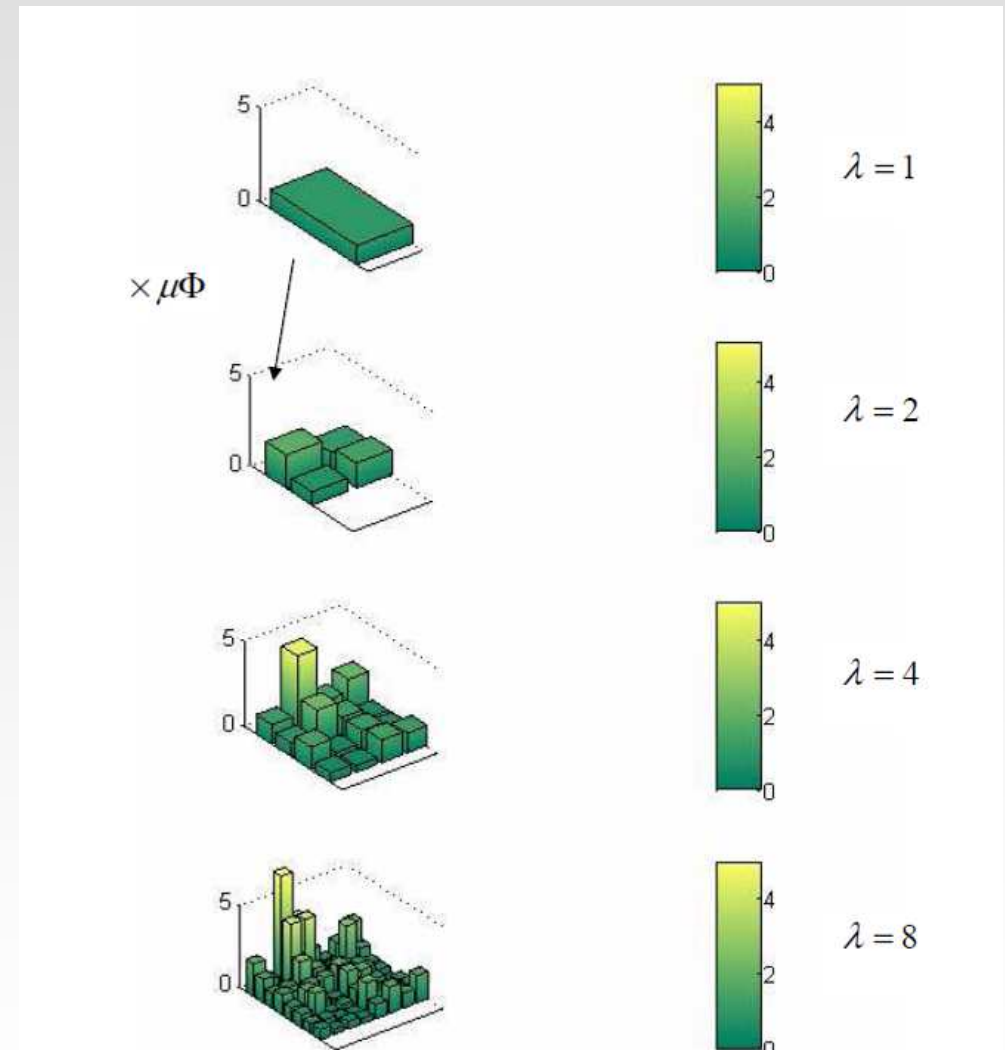
Stat. Moments of TRMM-PR reflectivities (Lovejoy et al., 2008)

# Multiplicative cascades

- Multifractal fields are built by a stochastic iterative approach

--> cascade  $\Phi$

- Possibly add a (scaling) **low-pass filter**
  - State variable is related to the positive quantity  $\Phi$  but is not  $\Phi$



Multiplicative cascade  $\Phi$ : i.i.d. multiplicative increments

# Statistical properties of multifractal fields

- Power-law energy spectrum

$$E(k) \approx k^{-\beta} \quad \beta = 1 - K(2) (< 1)$$

- **Statistical moments** of order  $q$  vary as a power-law of resolution  $\lambda$ :

$$\langle \Phi_\lambda^q \rangle \approx \lambda^{K(q)}$$

$q$  = statistics order ( $> 0$ , not necessarily integer)  
 $K(q)$  = moment scaling function

- **Filtered version** : Additional fractional integration (multiply by  $k^{-H}$  in Fourier space)
  - Pente spectrale  $\beta = 1 - K(2) + 2H$

# MU/FIF parameterization

- Universal multifractals (Schertzer & Lovejoy, 1987)

- Moment scaling function described by two parameters:

$$K(q) = \frac{C_1}{\alpha - 1} (q^\alpha - q)$$

$\alpha$  = multifractality parameter  $0 < \alpha < 2$

$C_1$  = inhomogeneity parameter (for « mean » intensities)  $0 < C_1 < D$

- FIF (Fractionnally Integrated Flux)

$\Phi$  is fractionnally integrated (at order  $H$ ), providing the FIF field  $X$ :

- Increments follow a scaling law:

$$\Delta X_\lambda = \Phi_\lambda \lambda^{-H} \quad (\text{in distribution})$$

- Analogous to Kolmogorov scaling law:

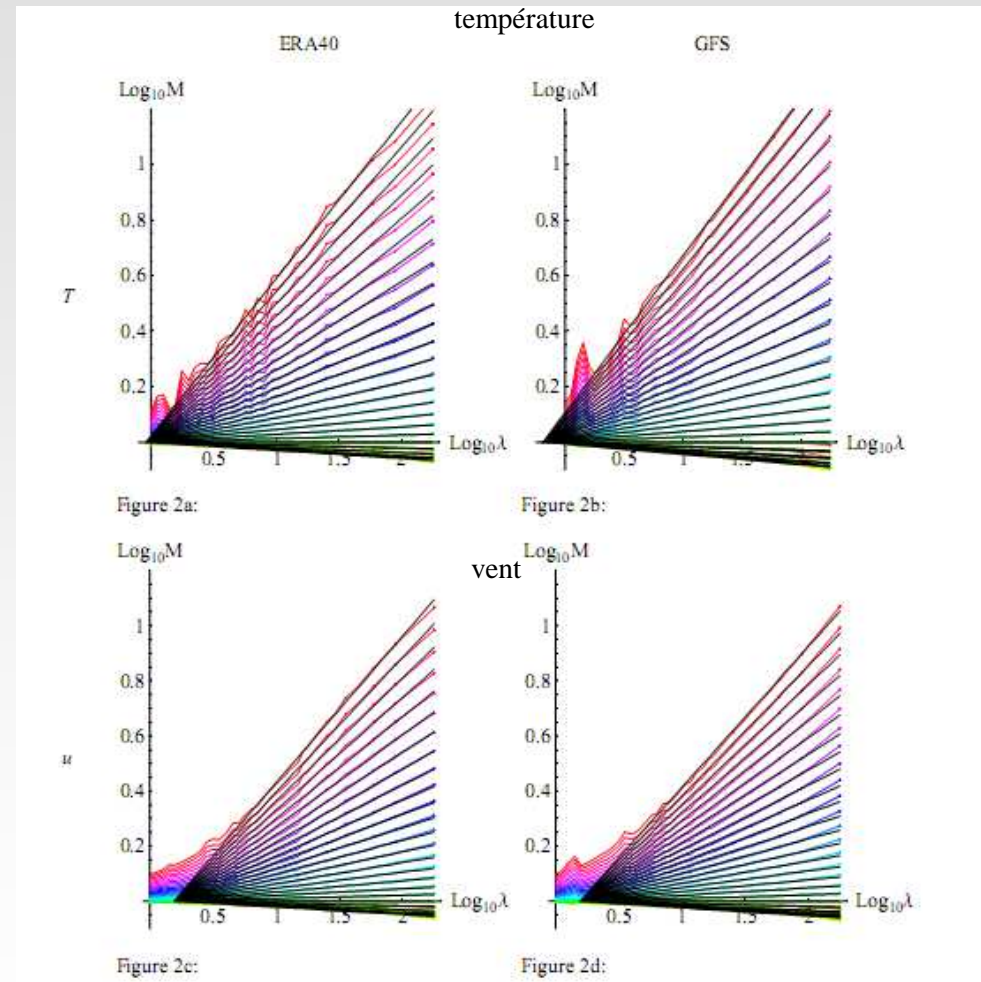
$$\Delta v_\lambda = \varepsilon_\lambda^{1/3} \Delta x^{-1/3}$$

$\Phi$  analogous to energy/variance dissipation



# Multifractals and AGCM?

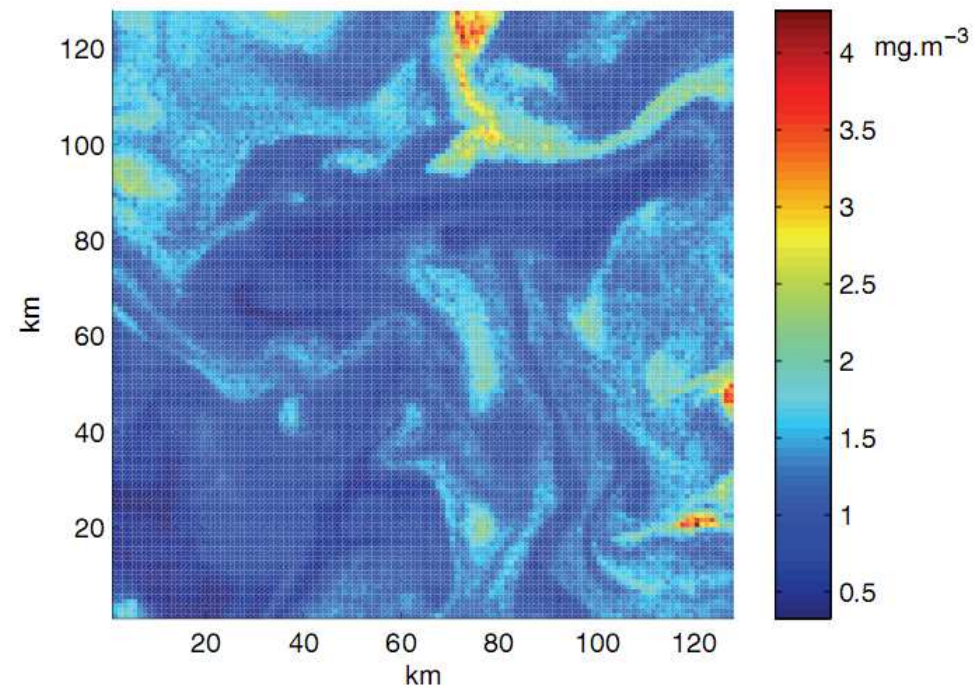
- Stolle et al. (2009) and Lovejoy et al. (2011):
  - ERA-40 and forecast AGCM
  
- Multifractal laws were found from planetary scales to  $\sim 1^\circ$
  
- Needs to be done for OGCM outputs ...





# Multifractals and ocean?

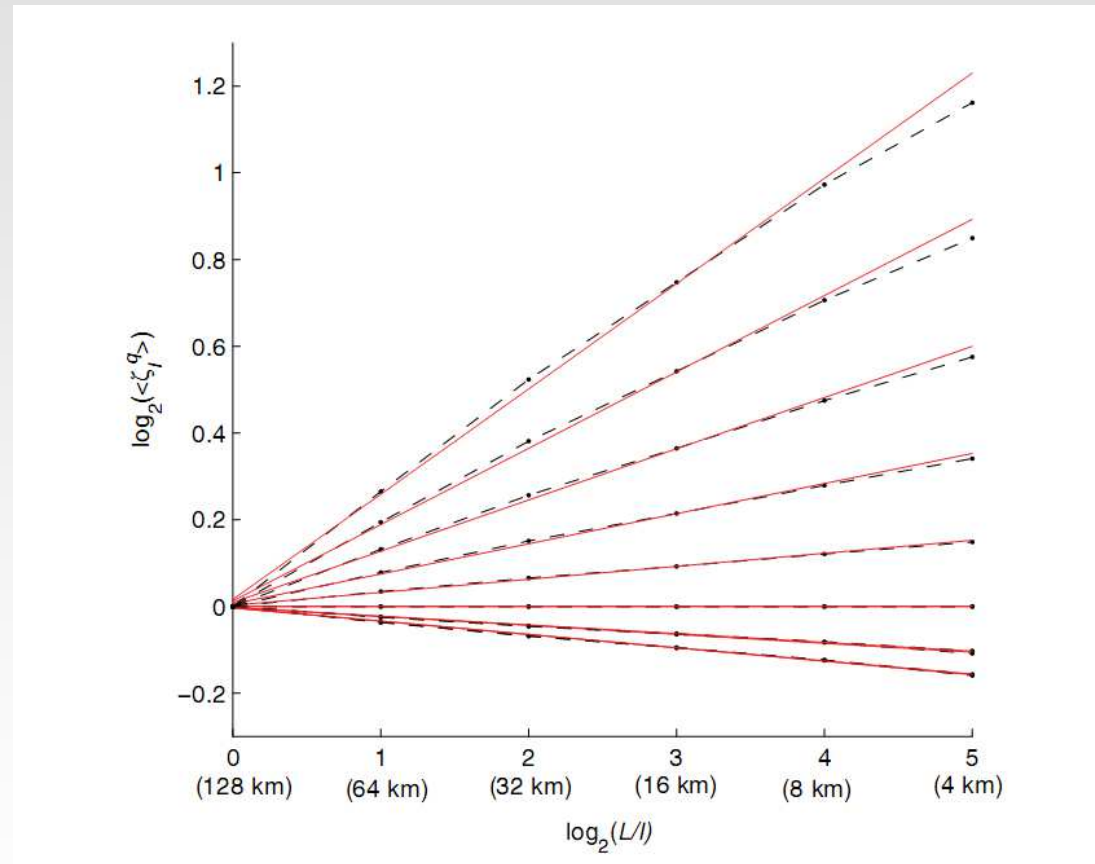
*What appears from satellite data (de Montera, Verrier, et al., 2011)*



**Fig. 2.** Example of a  $128 \text{ km}^2$  horizontal chlorophyll map (resolution  $1 \text{ km}^2$ ) extracted from the SeaWiFS local L2 product.

# Multifractals and ocean?

*What appears from satellite data (de Montera, Verrier, et al., 2011)*



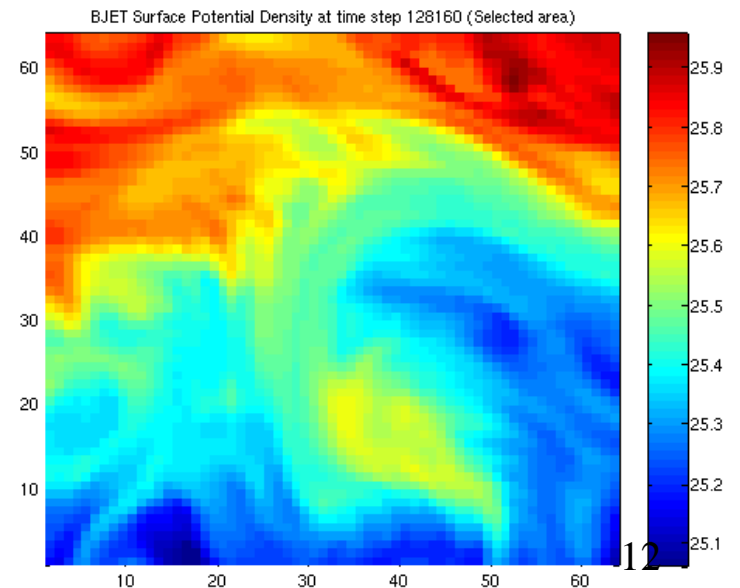
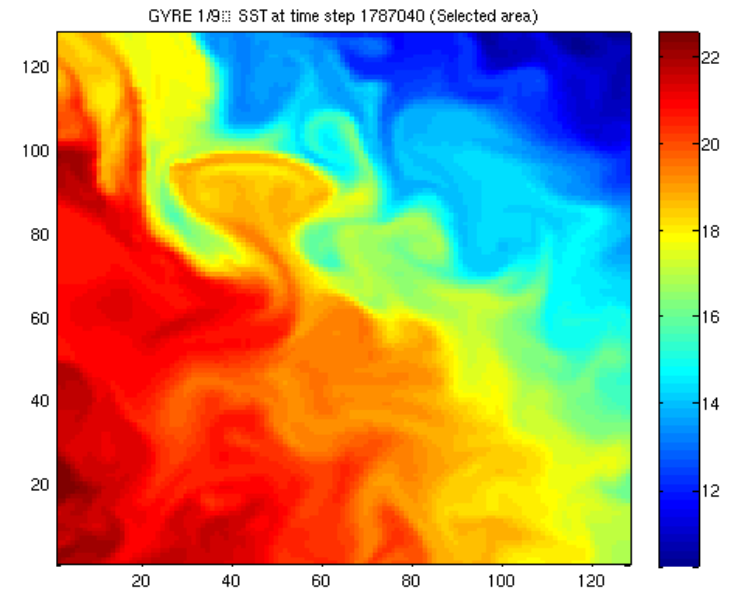
**Fig. 5.** Scaling of the statistical moments of the flux  $\zeta$  for the orders  $q=0, 0.1, 0.2, \dots, 2$ , with corresponding theoretical fits. Here,  $L$  corresponds to the largest scale of the SeaWiFS chlorophyll maps, i.e. 128 km. For each map, the flux was normalized to a mean value of 1.

# I) NEMO multi-order evaluation

- 1) Idealized simulations of NEMO
- 2) Determination of filtering exponent  $H$  ,  
deconvolution provides  $\Phi$
- 3) Study of  $\Phi$  moments for several positive orders,  
providing an estimate of  $\alpha$  et  $C1$

# NEMO simulation

- GYRE 1/9°
  - Spinup 50Y + study window 1Y
  - Surface data are considered
- BJET 5km
  - Zonal jet + baroclinic perturbation
  - Zonally periodic domain (~EEL)
  - Spinup 1Y + study 1Y



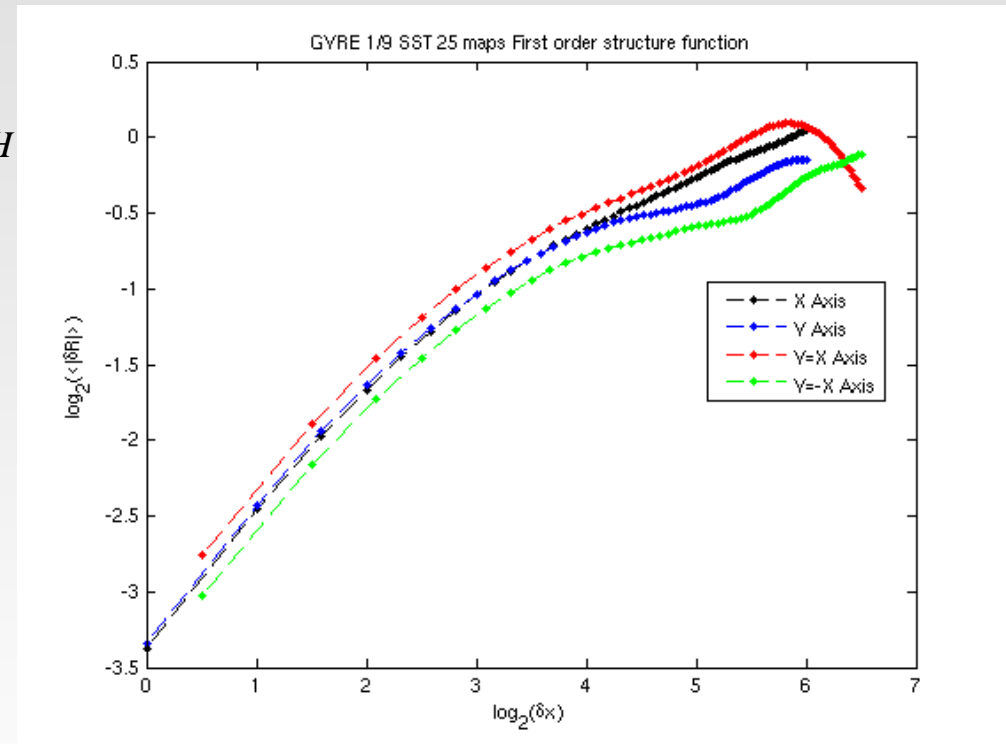
# Estimation of filtering parameter H

- Scaling of 1st order Kolmogorov structure functions :

$$\langle |SST(x + \delta x) - SST(x)| \rangle = \langle \Phi \rangle \delta x^H$$

$$\beta \sim 1 + 2H$$

- Piecewise scaling :
  - $H \sim 0.4$  for  $\delta x > 10 \times \text{gridstep}$
  - $H \sim 0.75$  for  $\delta x < 10 \times \text{gridstep}$
- Smoother variability at small scales, confirmed for other fields
- Physical regime ? Effective resolution problem ?

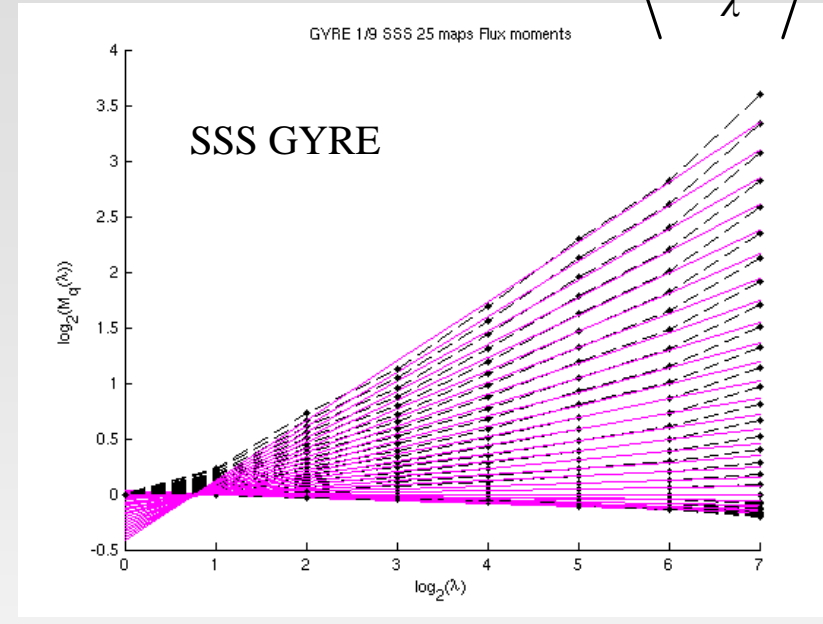
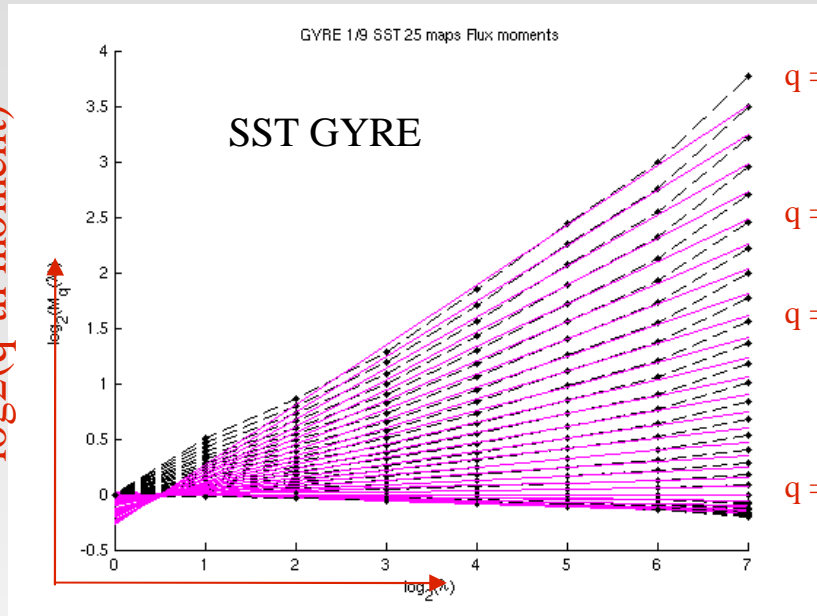


Absolute SST increments for the GYRE simulation, as a function of lag

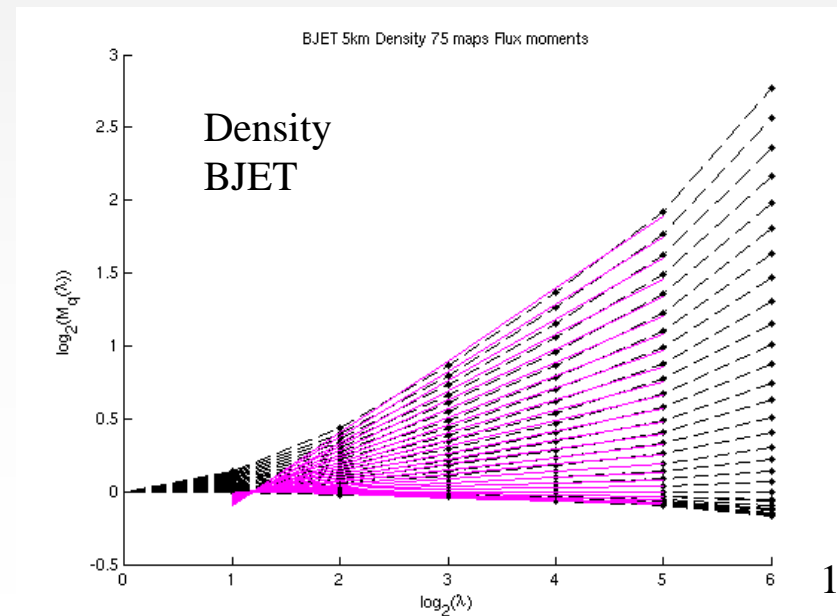
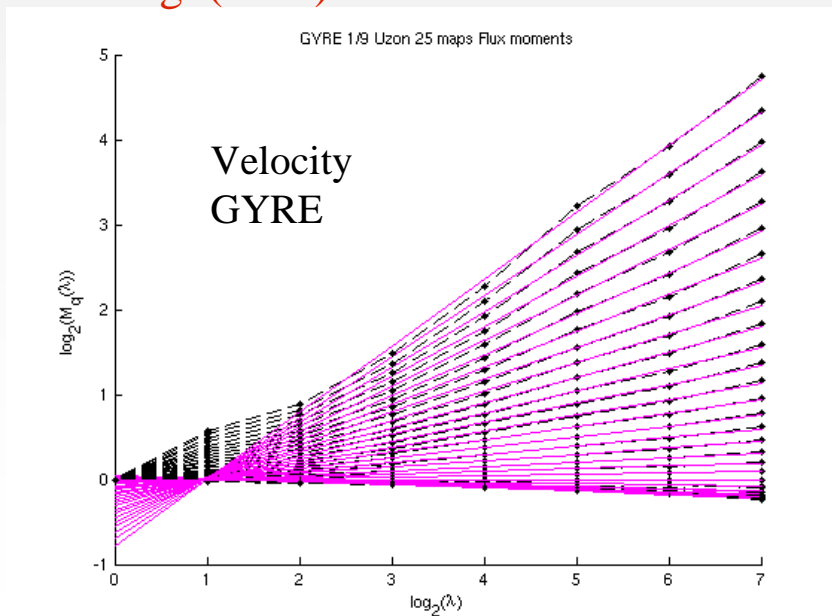
# Moments of $\Phi$ (log-log)

$$\langle \Phi_\lambda^q \rangle \approx \lambda^{K(q)}$$

log2(q-th moment)



log2(resol)



# Scaling exponents $K(q)$

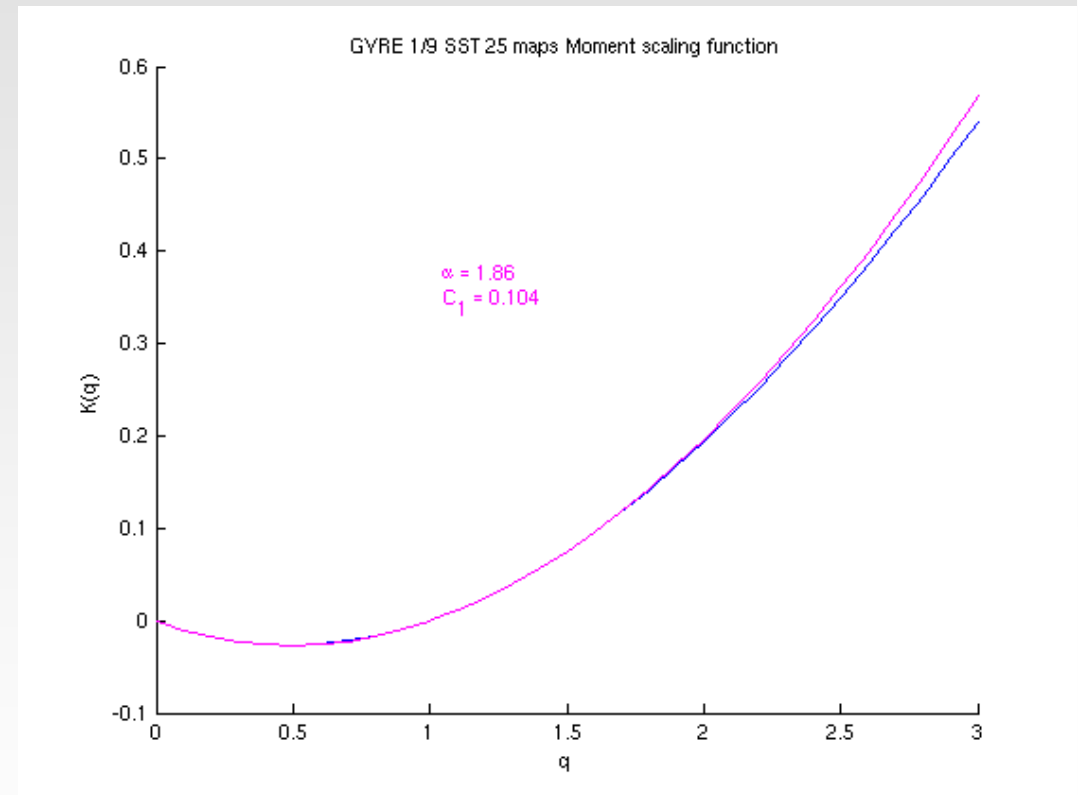
- Better scaling of  $\Phi$  statistics, down to  $O(2)$  x the gridstep
- Scaling is better for GYRE than for BJET
- Scaling exponents  $K(q)$  such that :

$$\langle \Phi_\lambda^q \rangle \approx \lambda^{K(q)}$$

- $K(q)$  accurately described by Schertzer-Lovejoy (universal) parameterization :

$$K(q) \sim C_1 / (\alpha - 1) (q^\alpha - q)$$

- Parameter values are coherent with oceanic empirical values



Values of  $K(q)$  for NEMO simulation and best-fit universal parameterization in the range  $0 < q < 2$



## II) Downscaling

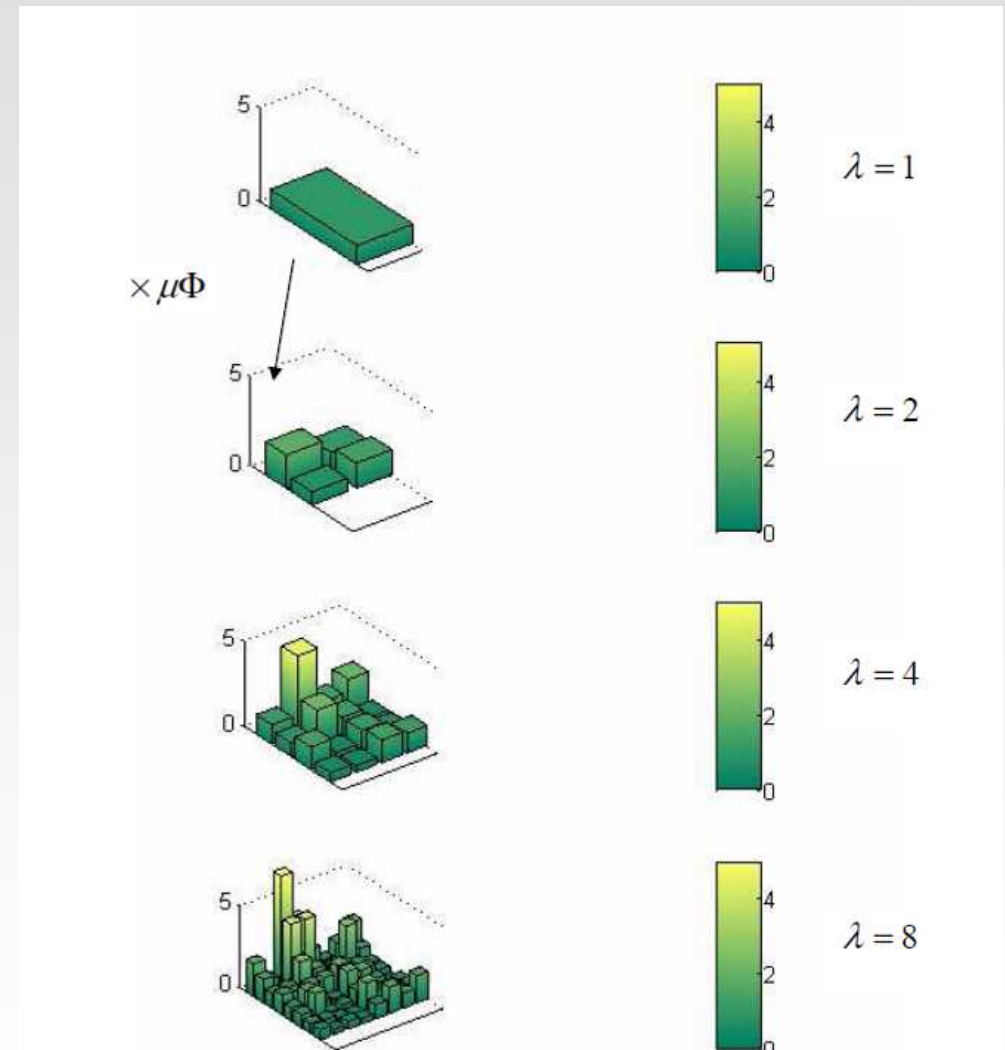
- **Purpose** : determining a description of unresolved variability that can convert low-resolution GCM outputs into higher-resolutions variables
  
- **Needed for important applications:**
  - Impacts of climate change (e.g., impact models need high-resolution precipitation inputs)
  - Downscaling atmospheric forcings used in high-resolution oceanic models
  - Helps to improve comparison between model outputs and data (satellite, in-situ/pointwise...) with different resolution

# Downscaling strategy

- **Dynamical** (regional GCM) vs **statistical** approaches
  - Dyn : correct location of extrema is expected, but computationnally demanding
  - Stat : correct probability distributions, error bars, higher gain in resolution ...
  
- Most existing statistical downscaling methods lack of physical justification
  - If correctly calibrated, it should work for converting a CDF from one specific scale to another
  - Representation of intermediate scales ? Fields structure ?  
Scaling symmetries?
  
- **Emerging approach : use multifractal cascades to simulate subpixel variability**
  - By construction, scaling symmetries are respected

# Multiplicative cascades (2)

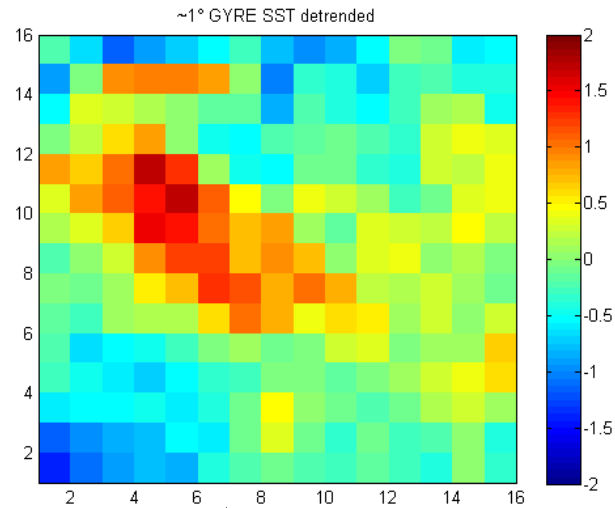
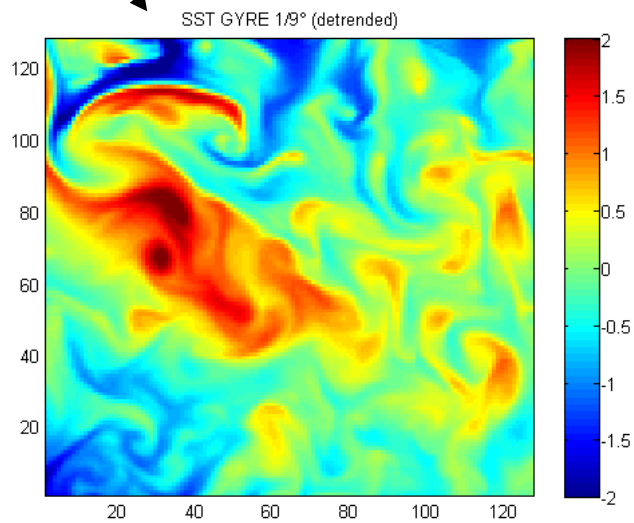
- Direct multiplicative cascades : downscale the multifractal flux  $\Phi$
- When  $H > 0$  : add a power-law filter
- Relatively inexpensive computationnally
- Well-suited for the simulation of an ensemble of high-resolution realizations



Multiplicative cascade  $\Phi$ : i.i.d. multiplicative increments

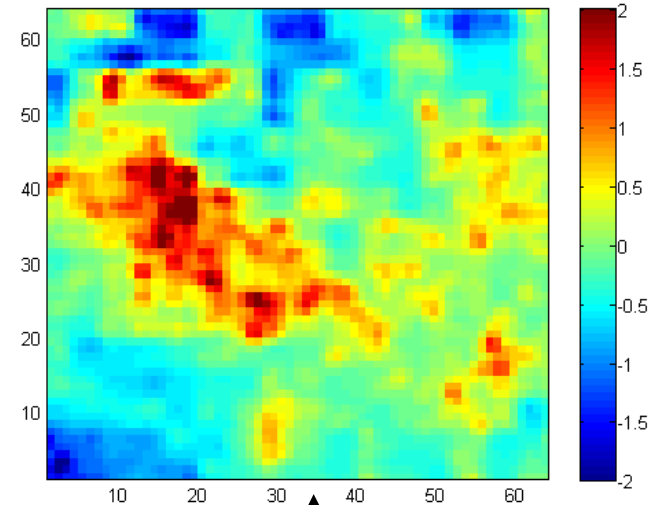
# An oceanic example (in progress)

1) GYRE  $1/9^\circ$   
Detrended SST



2) Aggregated  
detrended SST at  $1^\circ$

GYRE  $1^\circ \rightarrow 1/9^\circ$  downscaled  $a = 2$   $H = 0.45$



3) Downsampled data at  $1/9^\circ$

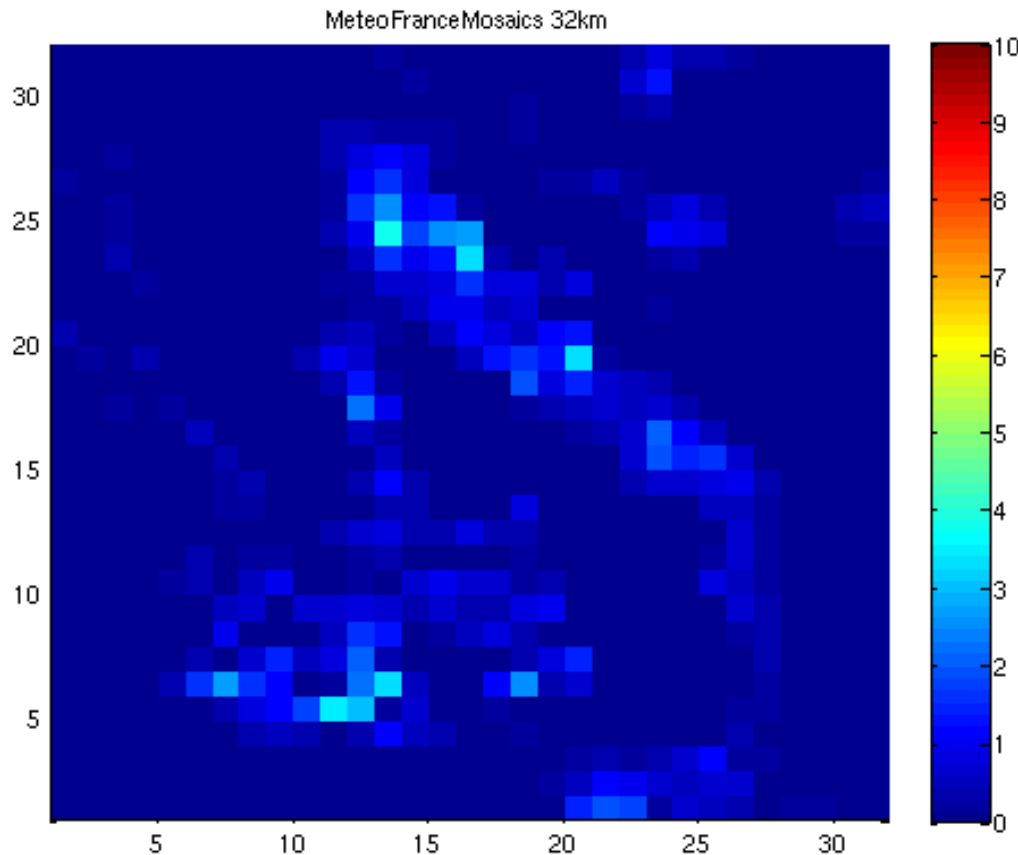
# A more developed example on rainfall data

- Rainfall is piecewise multifractal with specific parameters
- Real data: Radar mosaics are considered (1000x1000 km at 1km resolution)
- Scaling ranges 32-8km et 8-1km

# Inferring sub-pixel variability

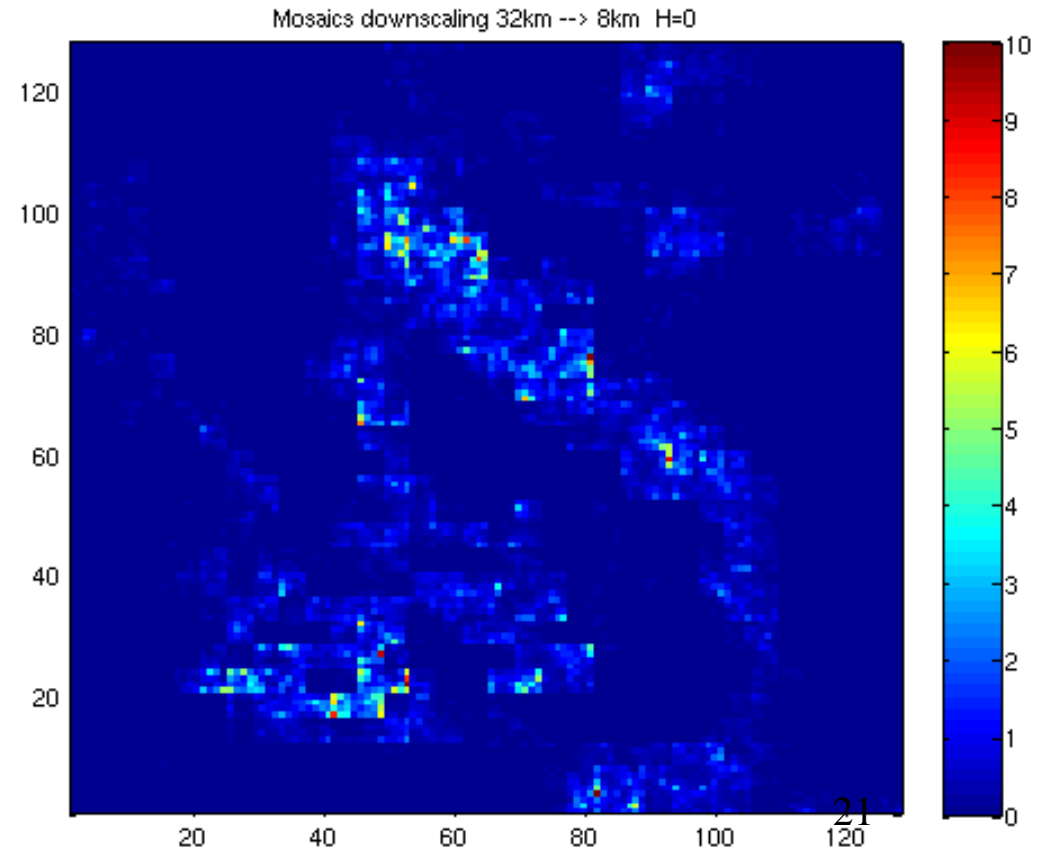
- Possibility to simulate stochastic sub-pixel variability by extrapolating multifractal scaling laws...
- By construction, accurate retrieval of the CDF at multiple scales

Rainfall composite data aggregated  
at 32 km scale



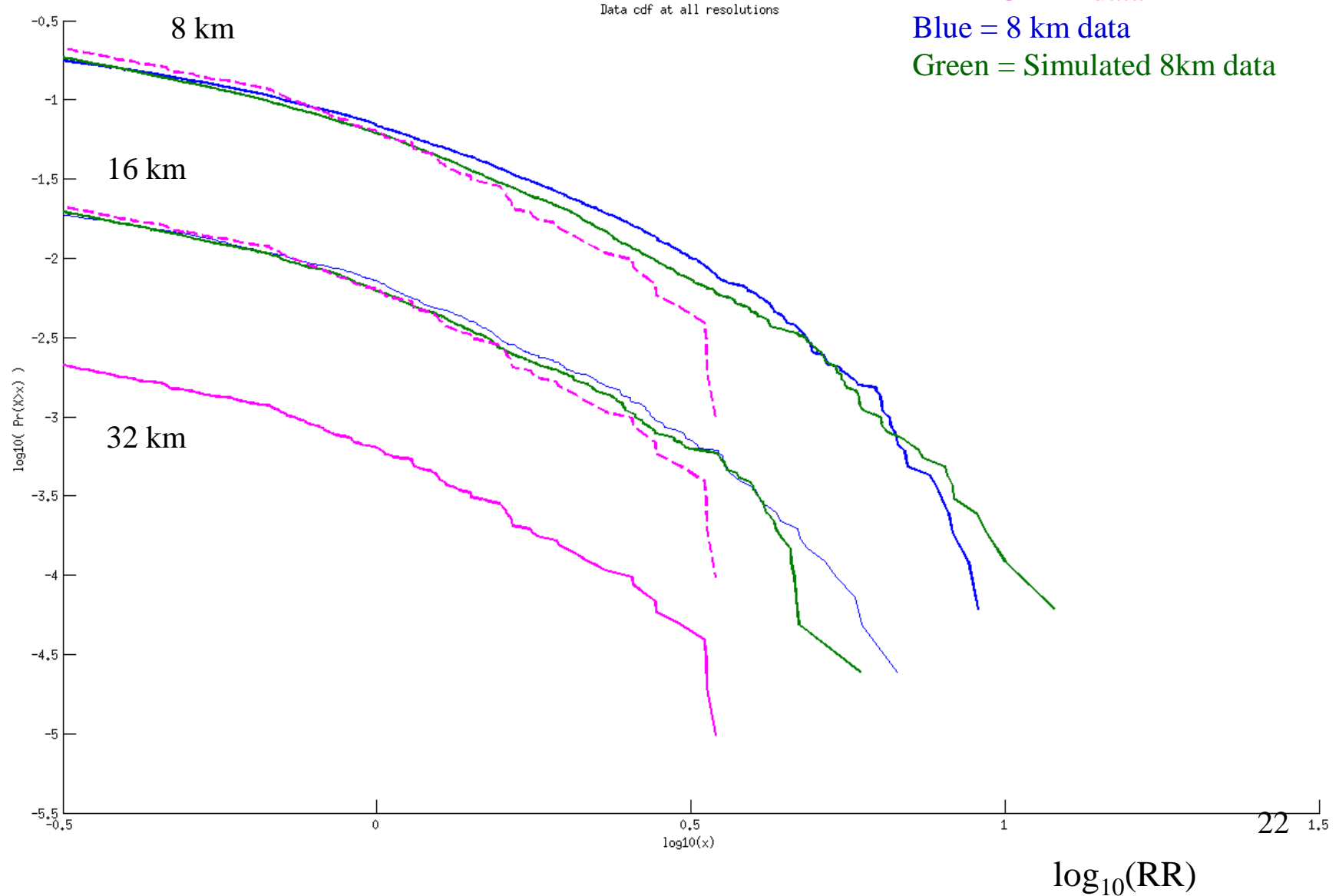
*MeteoFrance Mosaics rain rate (26/03/2008)*

Rainfall composite data  
disaggregated at 8 km scale



# Multifractal downscaling 32-8 km (example on 1 map)

$\log_{10}(\Pr(RR > x))$

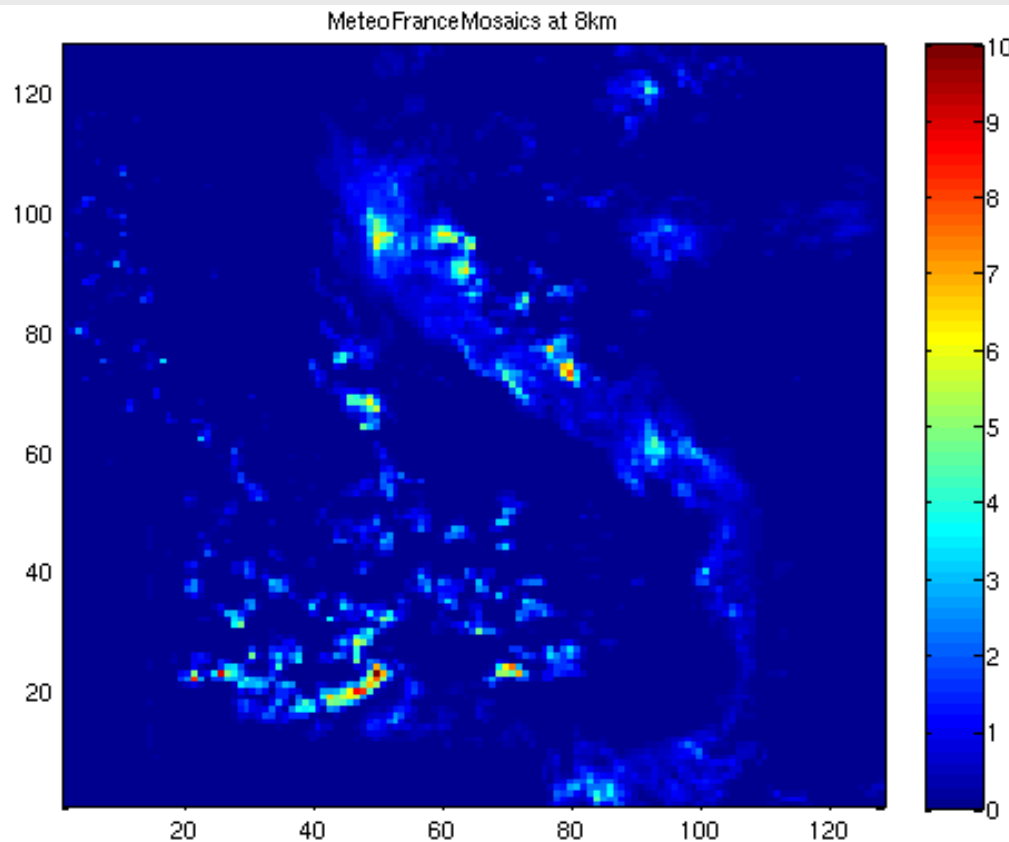




# Inferring sub-pixel variability

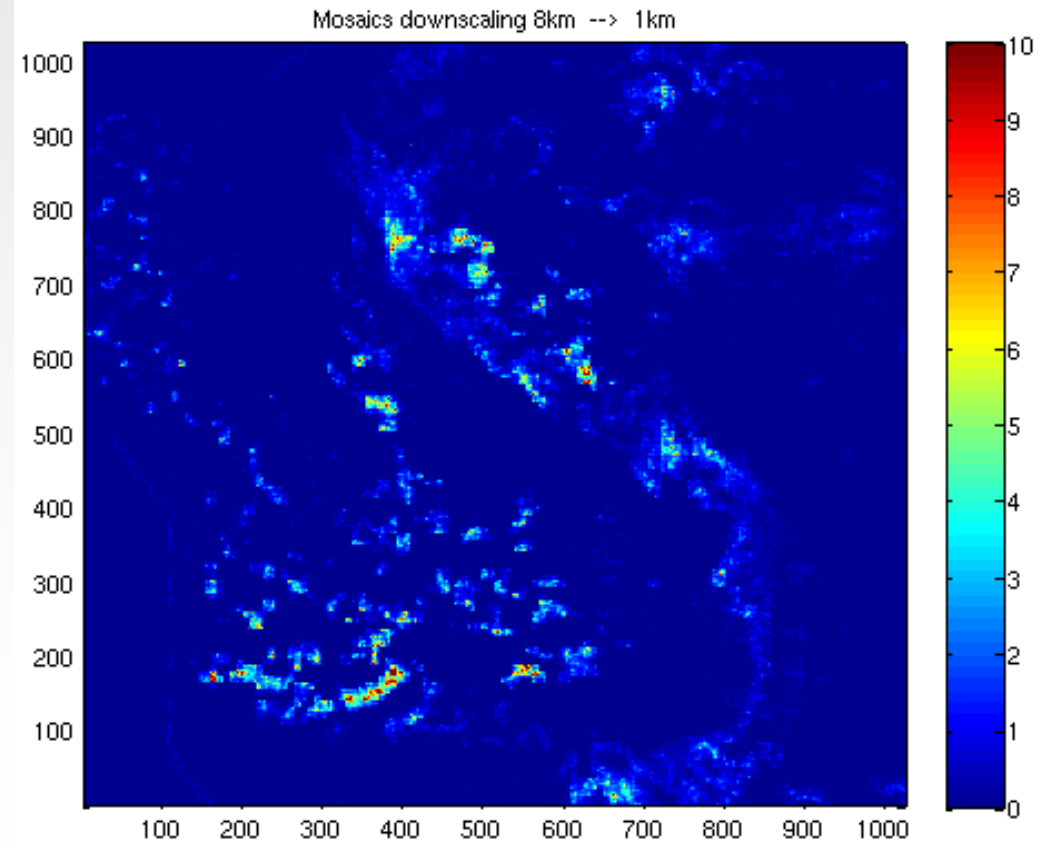
- Possibility to simulate stochastic sub-pixel variability by extrapolating multifractal scaling laws...
- By construction, accurate retrieval of the CDF at multiple scales

Rainfall composite data aggregated at 8 km scale

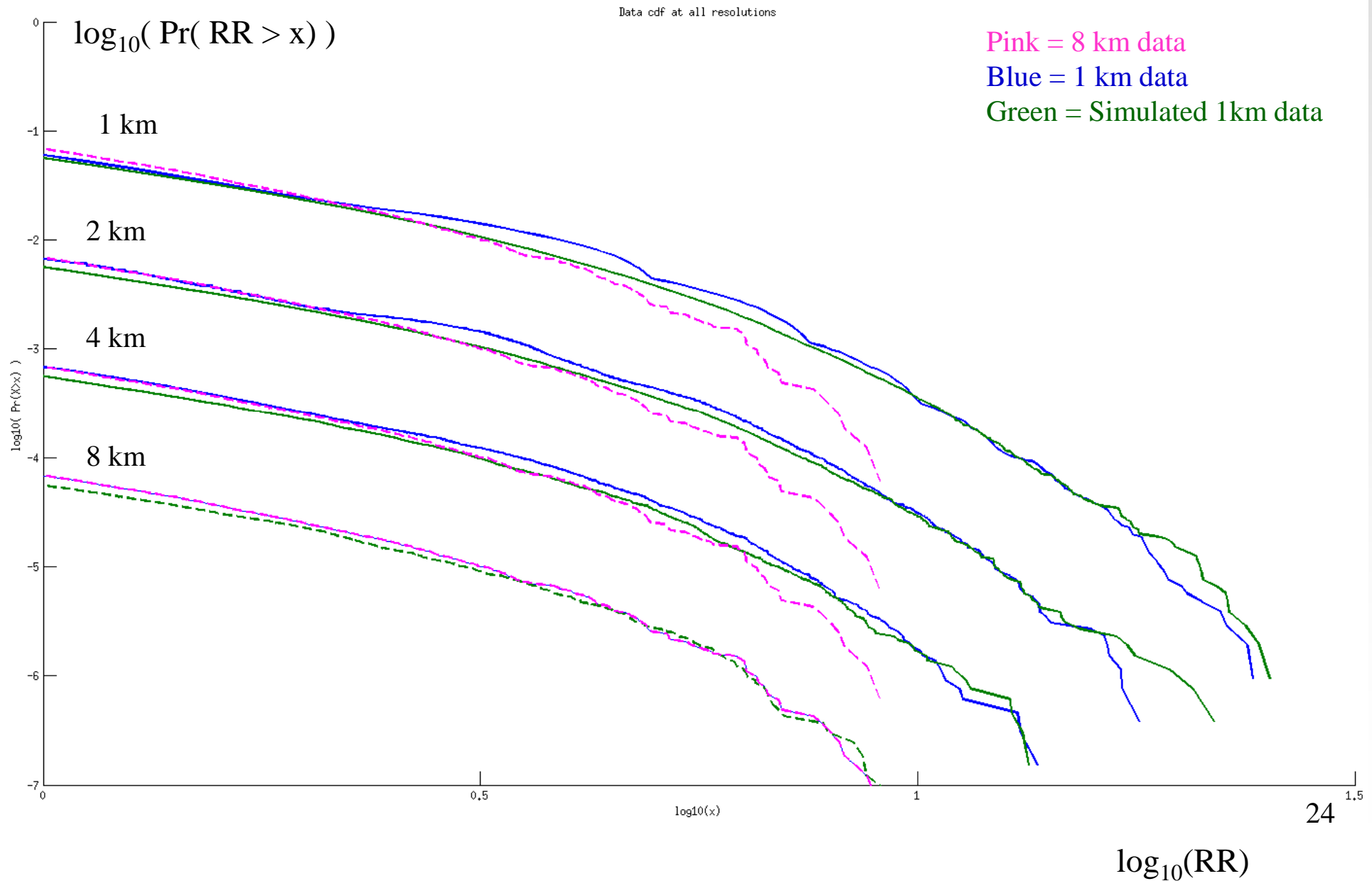


*MeteoFrance Mosaics rain rate (26/03/2008)*

Rainfall composite data disaggregated at 1 km scale



# Multifractal downscaling 8-1 km (example on 1 map)



### III) Variational control of diffusion parameters in NEMO

- Variational approach
- Twin experiments (obs = other simulation)
- Two trajectories with different resolutions
  - High resolution trajectory provides « observations »
  - Low resolution trajectory : viscosity and diffusivity are controlled as function of space point  $f(x,y,z)$
- **Purpose** : study the link between eddy viscosity and local properties of the flow

# YAO

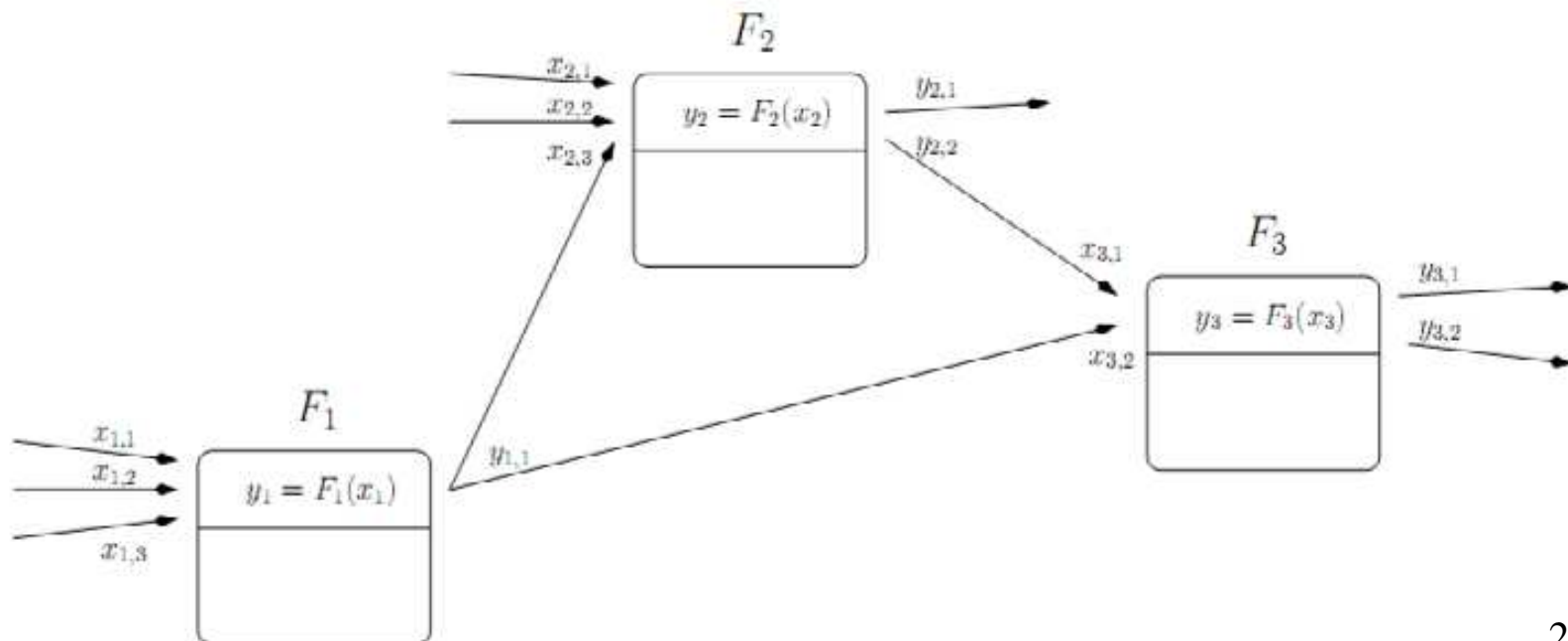
- Software dedicated to variational data assimilation
- Modular description of a numerical model
- Automatic computation of the adjoint model from elementary jacobians

[www.locean-ipsl.upmc.fr/~yao/](http://www.locean-ipsl.upmc.fr/~yao/)

# YAO

## Direct model

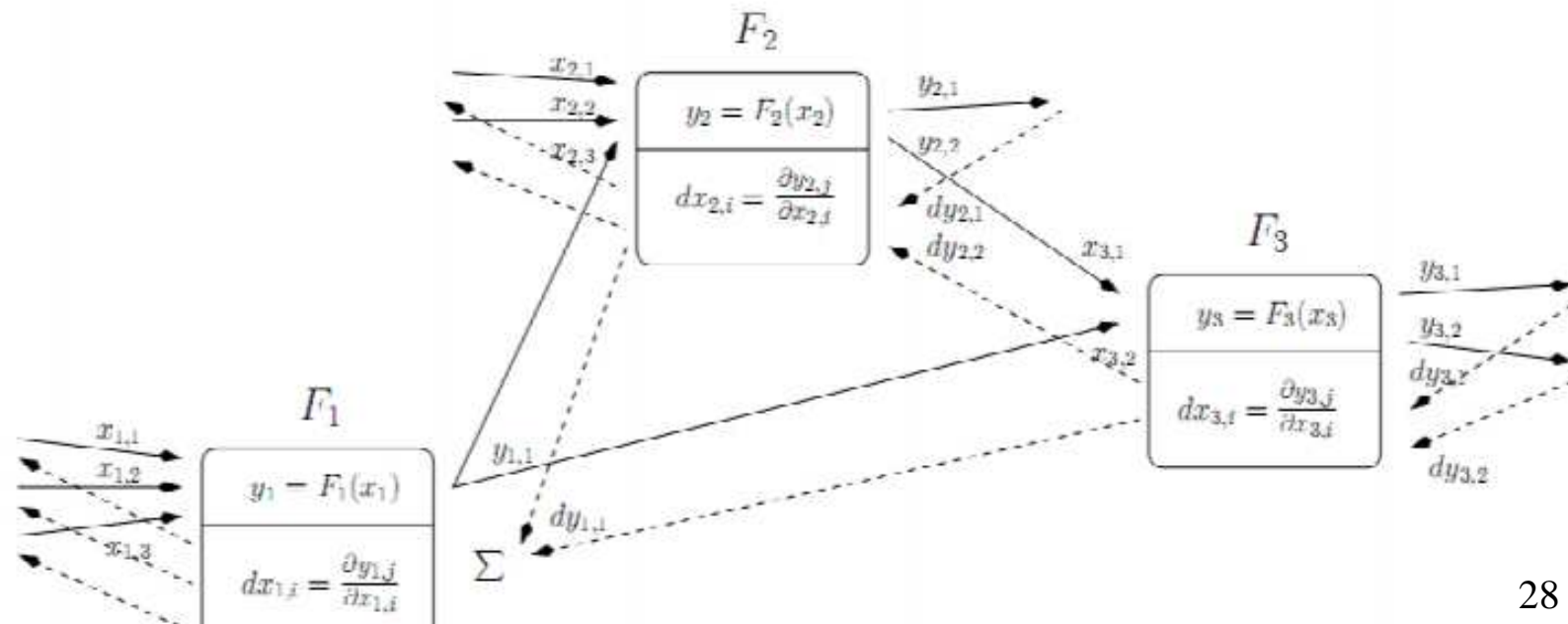
Passing through the graph in a topological order we calculate the direct model : **forward algorithm**



# YAO

## Adjoint model

Passing through the graph in a reverse topological order we calculate the adjoint model : **backward algorithm**

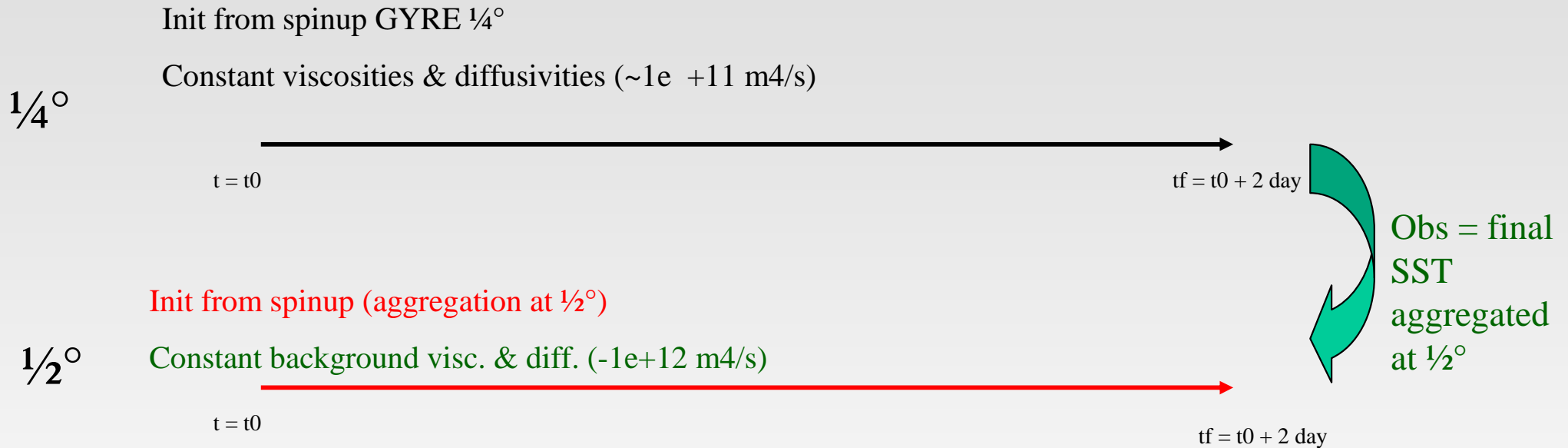


# NEMO-YAO configuration

- Version of NEMO coded within YAO assimilation software
  
- “Translation” of GYRE idealized configuration
  - Available resolutions :  $1^\circ$ ,  $1/2^\circ$  et  $1/4^\circ$
  - 4th order diffusive scheme (2<sup>nd</sup> order also available)
  
- Trajectories initialized by a 30Y GYRE (fortran) spinup



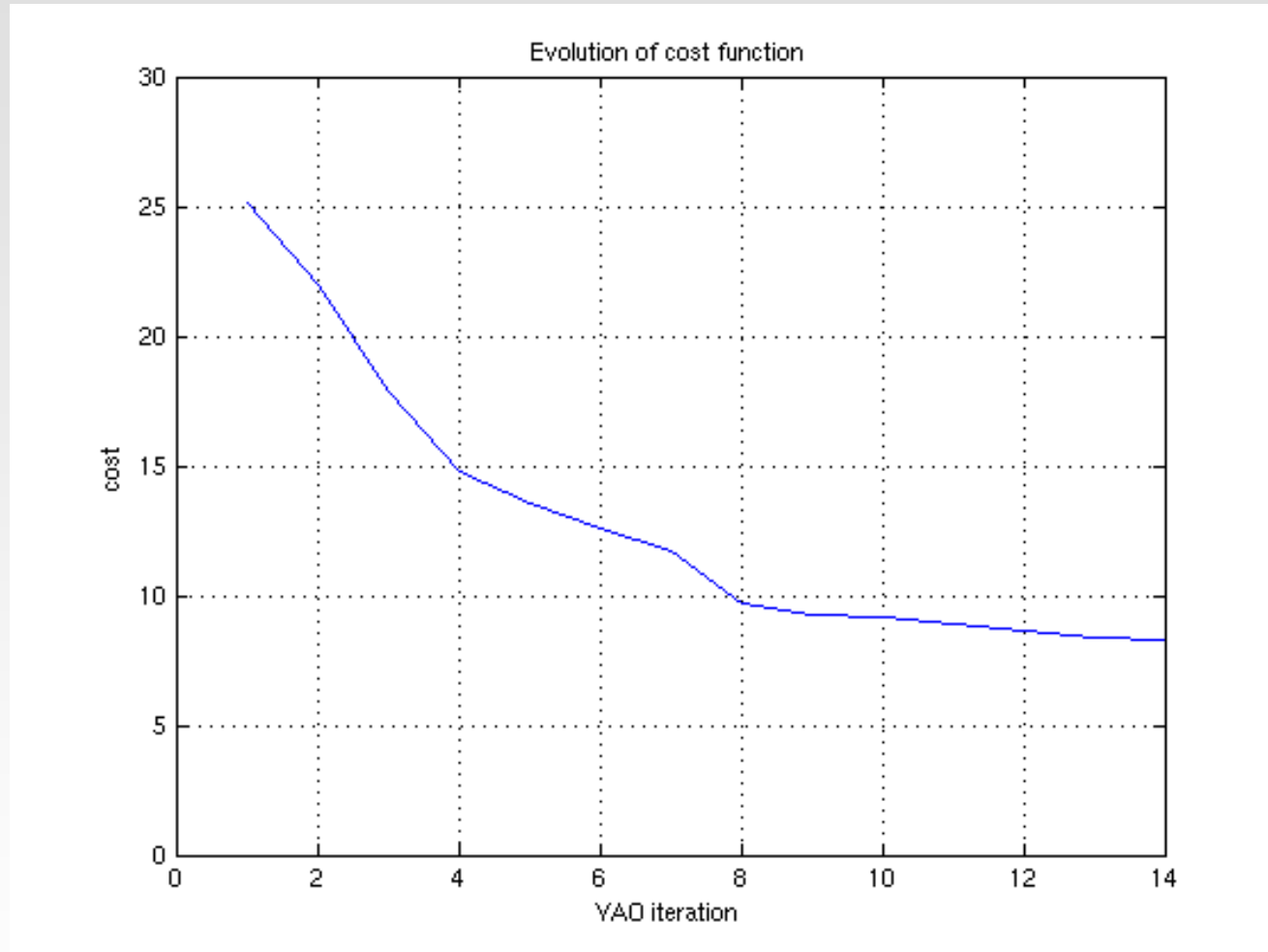
# Two-resolution experiment



Cost function minimization (YAO)

Control parameters = spatial fields of bilapacian viscosities & diffusivities

# Minimisation of $J(\text{diffusivity, viscosity})$

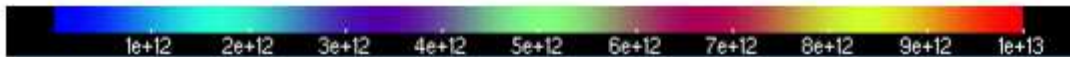
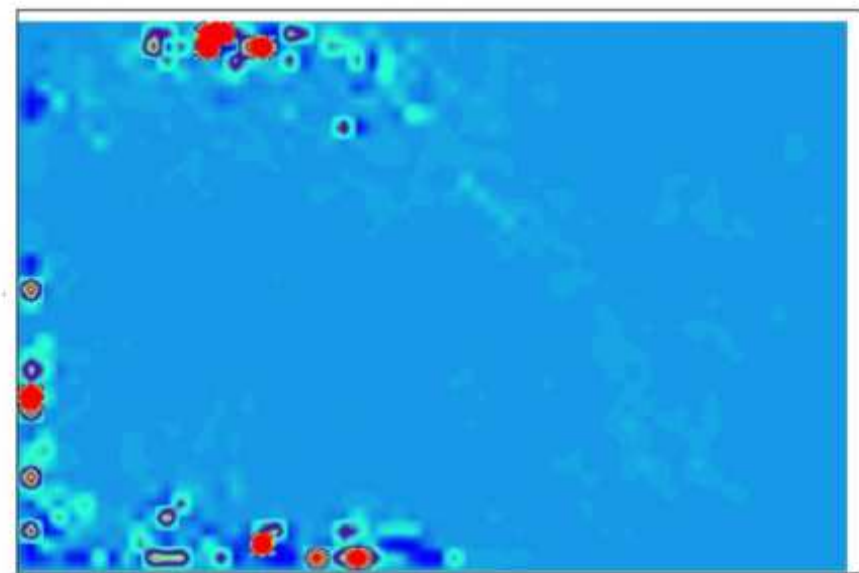
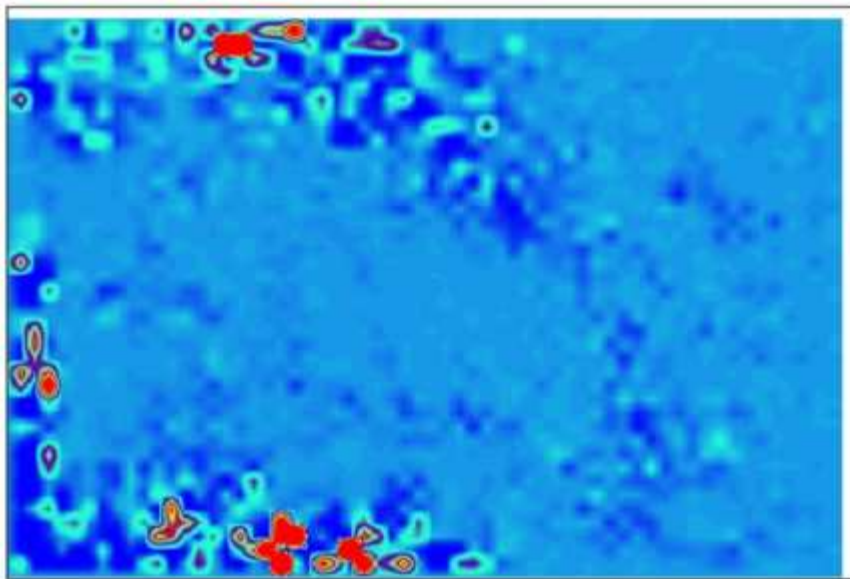


Gain of a factor 3 by controlling 3D diffusive coefficients only  
(future improvements expected by adding control of initial conditions)

# Surface diffusive coefficients

|Surface Diffusivity|

|Surface Viscosity|

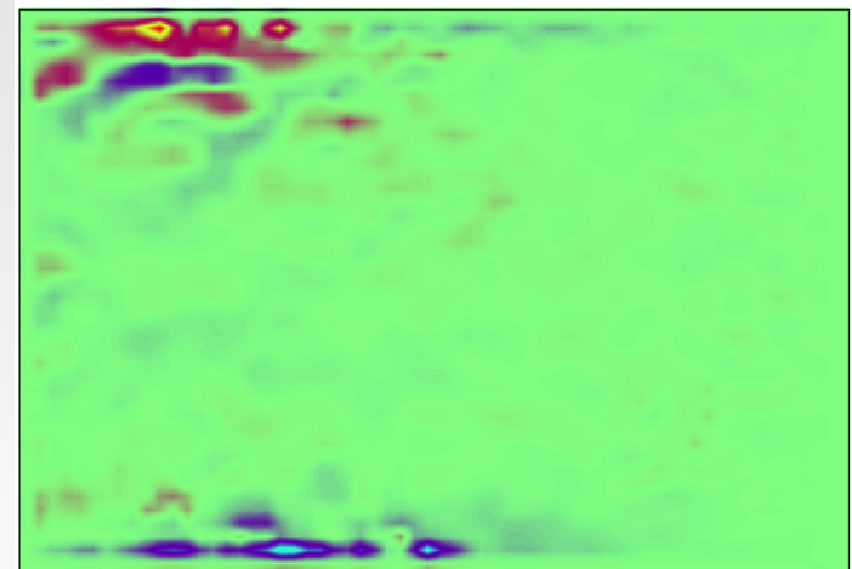
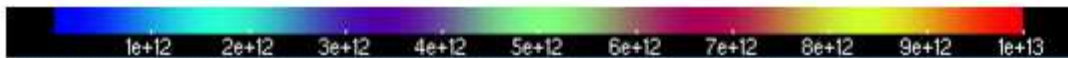
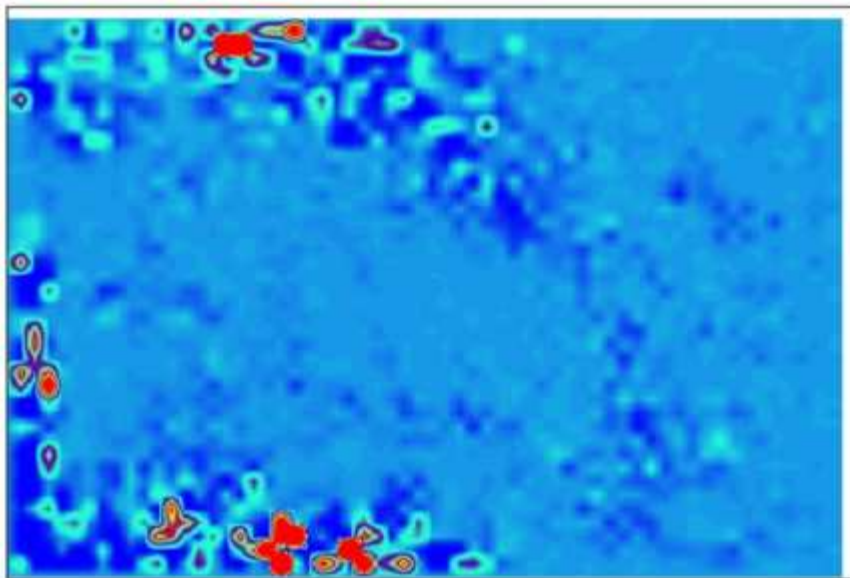


Background value =  $1.0e+12$  m<sup>4</sup>/s (background color)

# Surface patterns localization

|Surface Diffusivity|

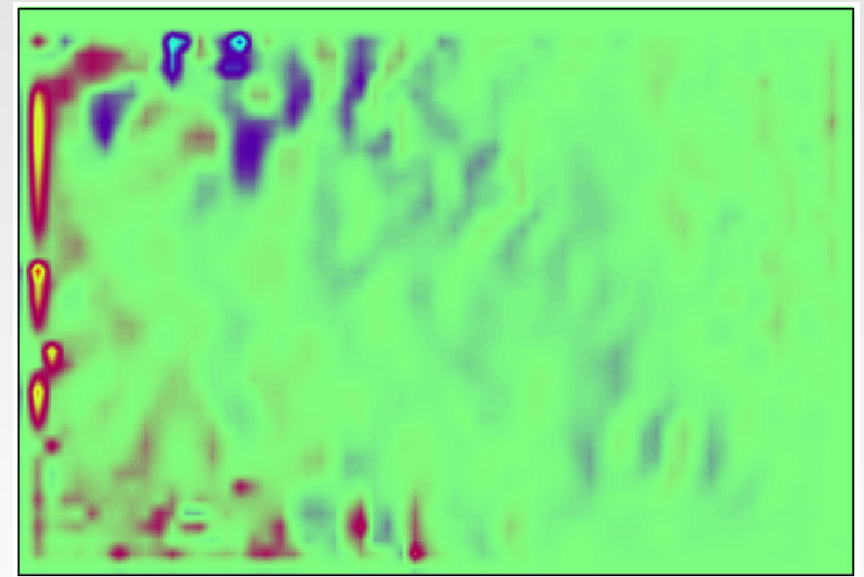
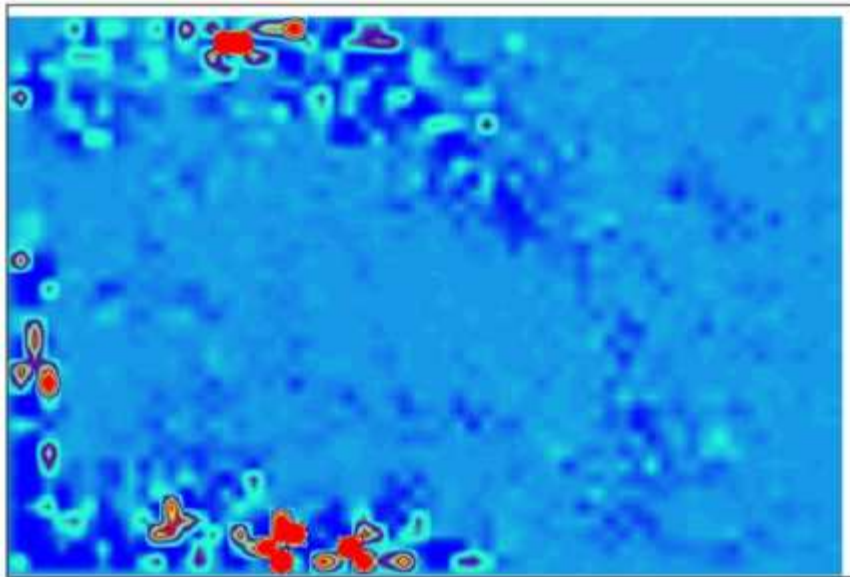
Velocity (u component) snapshot



# Surface patterns localization

|Surface Diffusivity|

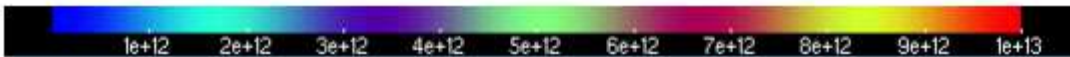
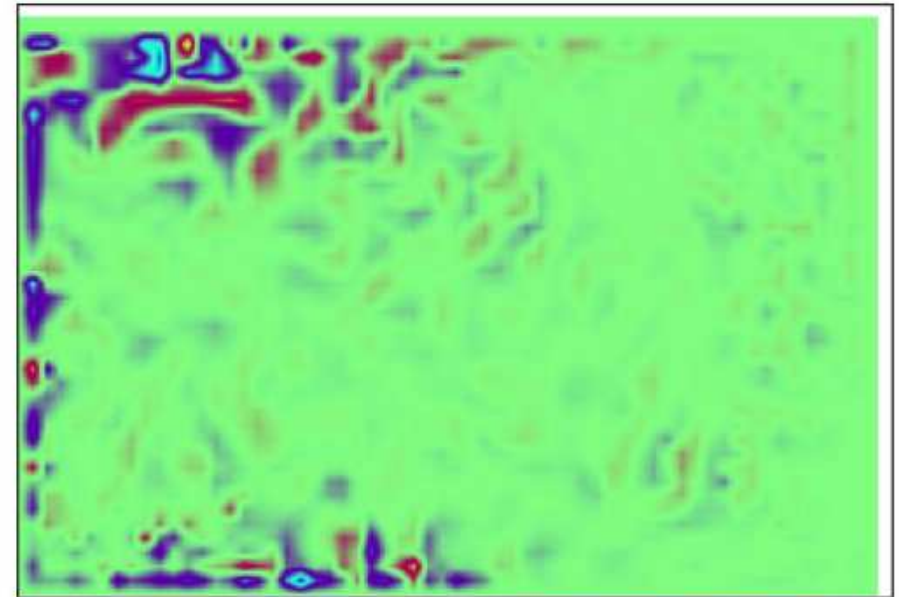
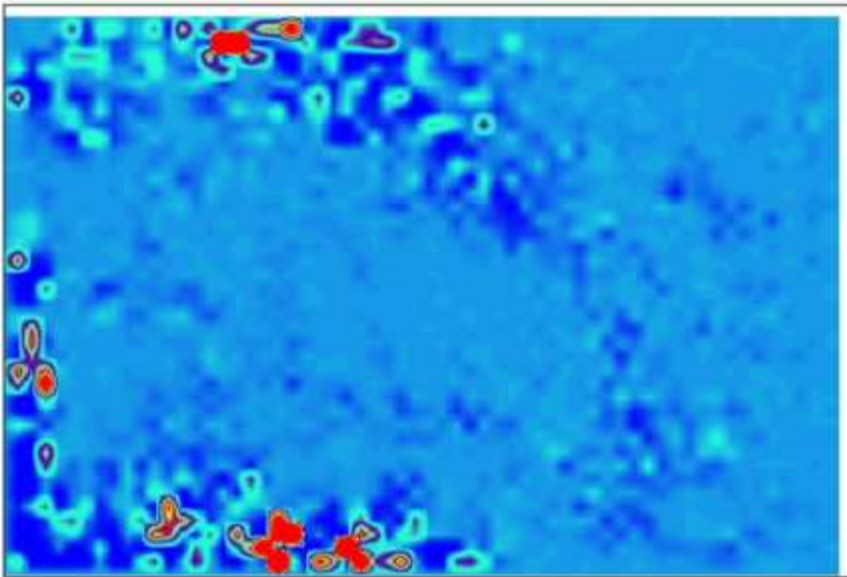
Velocity (v component) snapshot



# Surface patterns localization

|Surface Diffusivity|

Vorticity snapshot

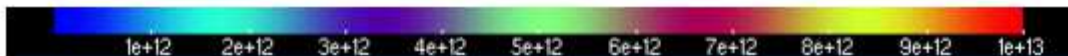
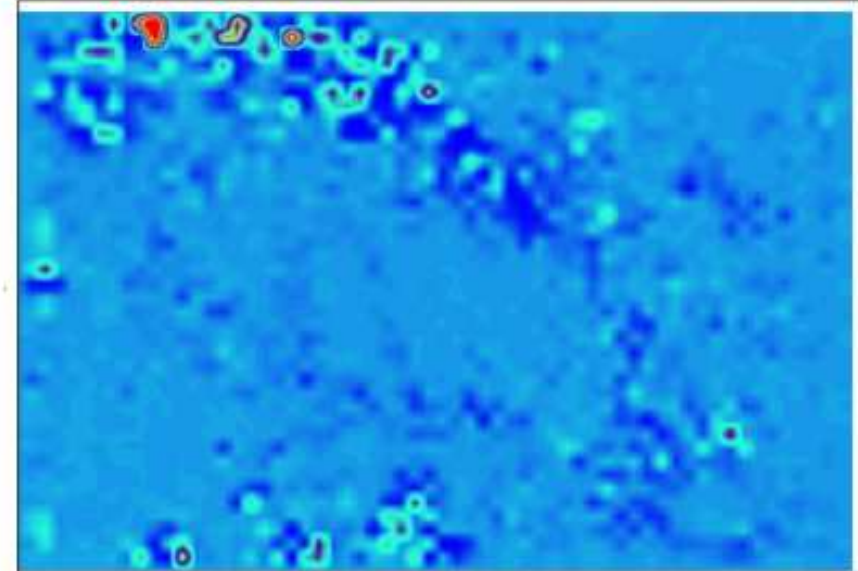
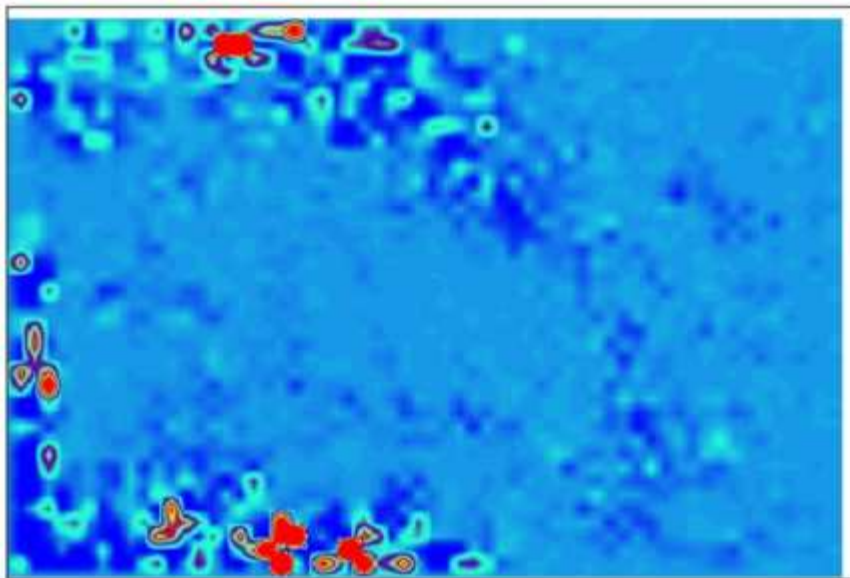




# Vertical coherence of patterns

|Surface diffusivity|  $k=0$

|Diffusivity| at level  $k=5$  (60 m depth)



Significant patterns even at level  $k = 10$  (135 m)  
 Note that only SST data have been assimilated



# Conclusions

- **Multifractal scaling properties:**
  - A statistical tool for OGCM/AGCM evaluation
  - Multi-order = generalizes spectral tools
  
- **Downscaling:**
  - Multifractal properties provide information on CDF transformations when changing resolution
  - Relatively inexpensive method, parameterizable
  - Respects fundamental symmetries of the flow
  
- **NEMO-YAO:**
  - Feasibility study for controlling diffusive parameters and comparing with local properties of the flow

# Perspectives

- **Multifractal scaling properties:**
  - To be validated in available higher-resolution NEMO simulations ( $1/54^\circ$ )
  - Comparison with ROMS
  
- **Downscaling:**
  - Modifications needed for simulating accurate filamentary structures
  - Method not specific to SST or rainfall, could be adapted for downscaling wind forcings
  
- **NEMO-YAO:**
  - Larger assimilation window, higher resolutions
  - More complex diffusive schemes



# Questions?

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