# Reconstruction of the 3D mixed-layer dynamics from space using satellite data, numerical insight

**Aurélien Ponte, Patrice Klein**

**An illustration of our approach at LPO:**
- **start with high resolution numerical solutions**
- **understand the dynamics that takes place**
- **develop methods in order to reconstruct this dynamics with data (satellite and in situ)**

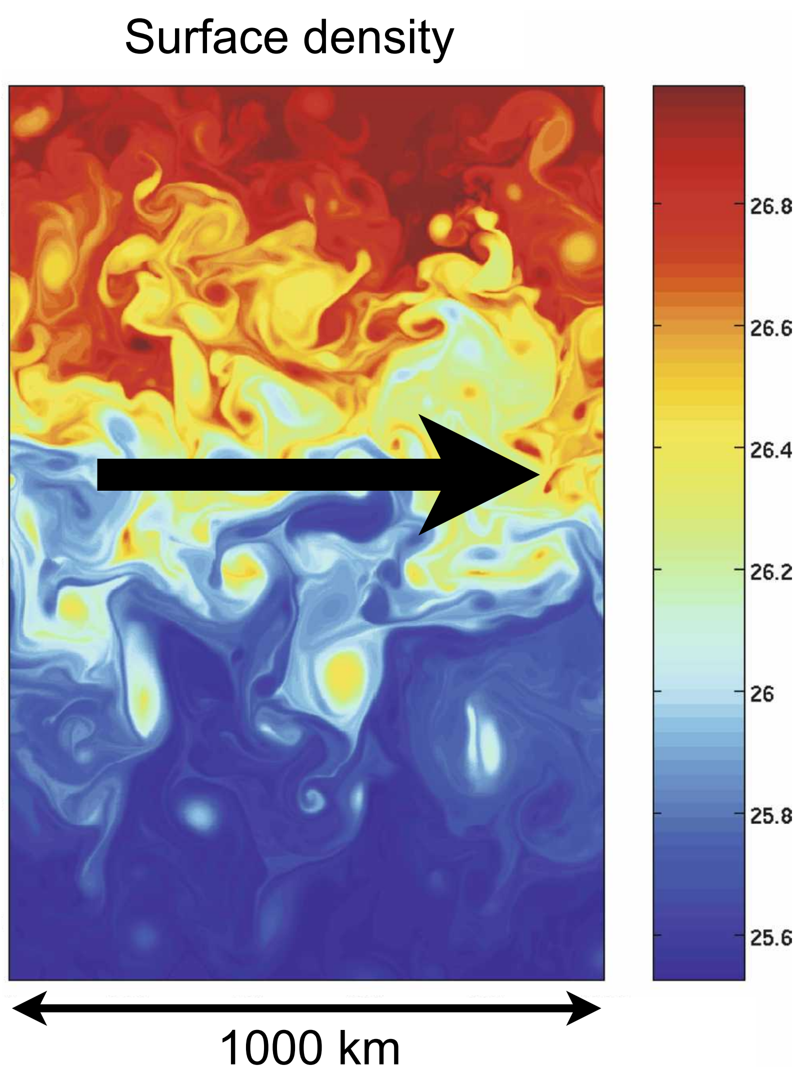
# A numerical testbed of ocean turbulence

**Numerical experiments of a mid-latitude baroclinically unstable jet**

**Zonally periodic channel**
**Relaxation toward unstable initial state**

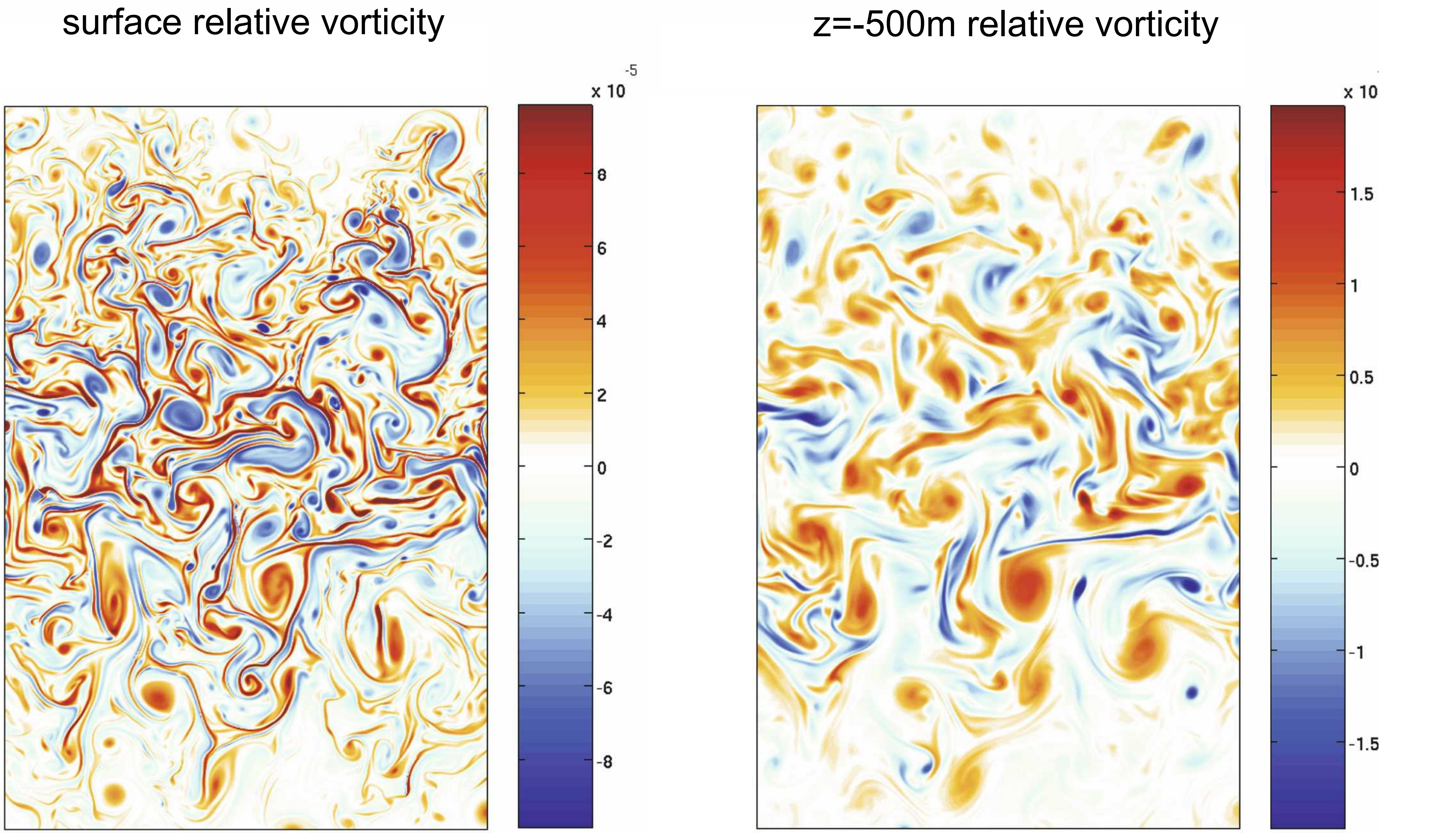
**dx=2 km, Nz=100**

**Regime surface intensified turbulence**
**Stirring of surface density**




*(Klein et al. 2008)*

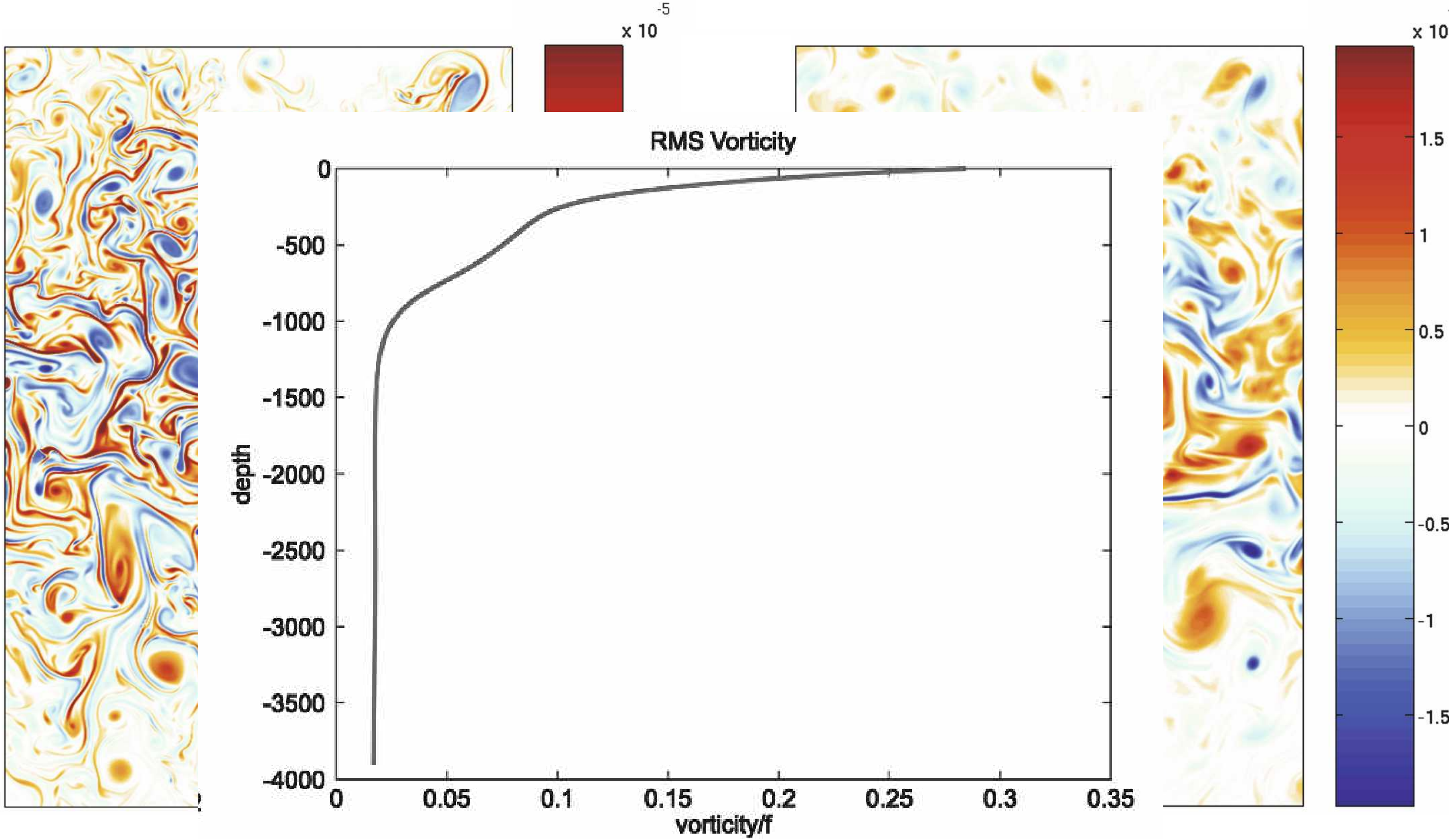
# Surface intensified turbulence



# Surface intensified turbulence

surface relative vorticity

z=-500m relative vorticity

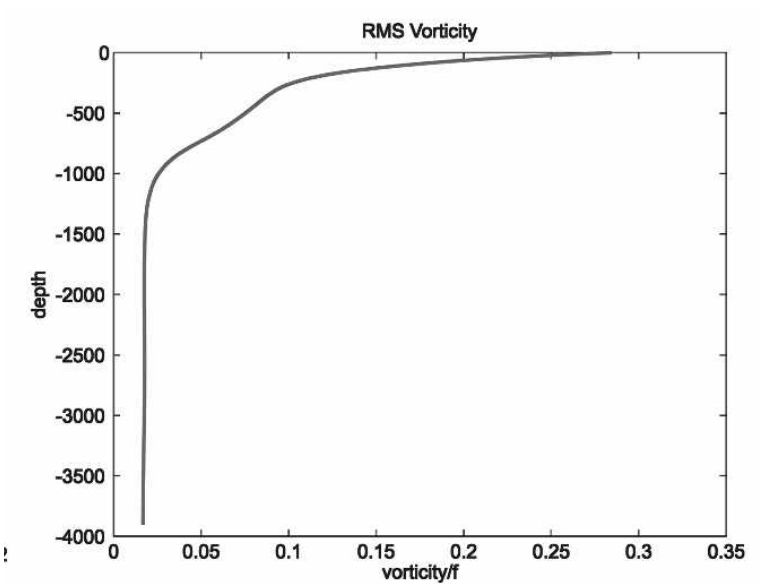


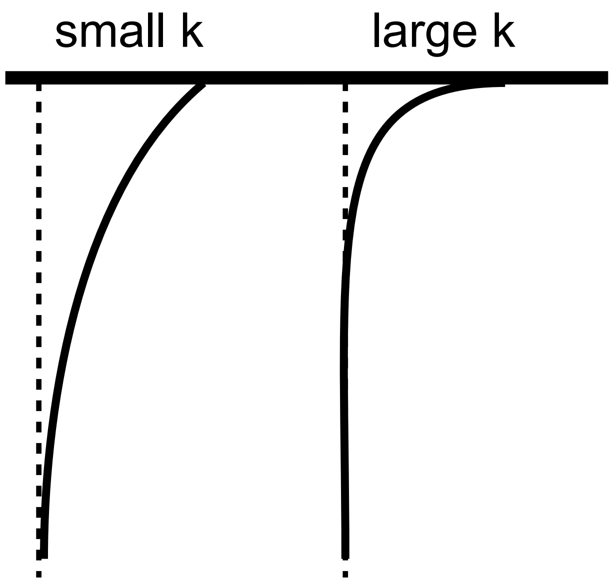
# Surface Quasi-Geostrophy

**Small Ro(=U/fL) approximation of the dynamics in the presence of surface density anomalies**
**In the present regime of turbulence, you can verify that a good approximation of the horizontal streamfunction is given by:**

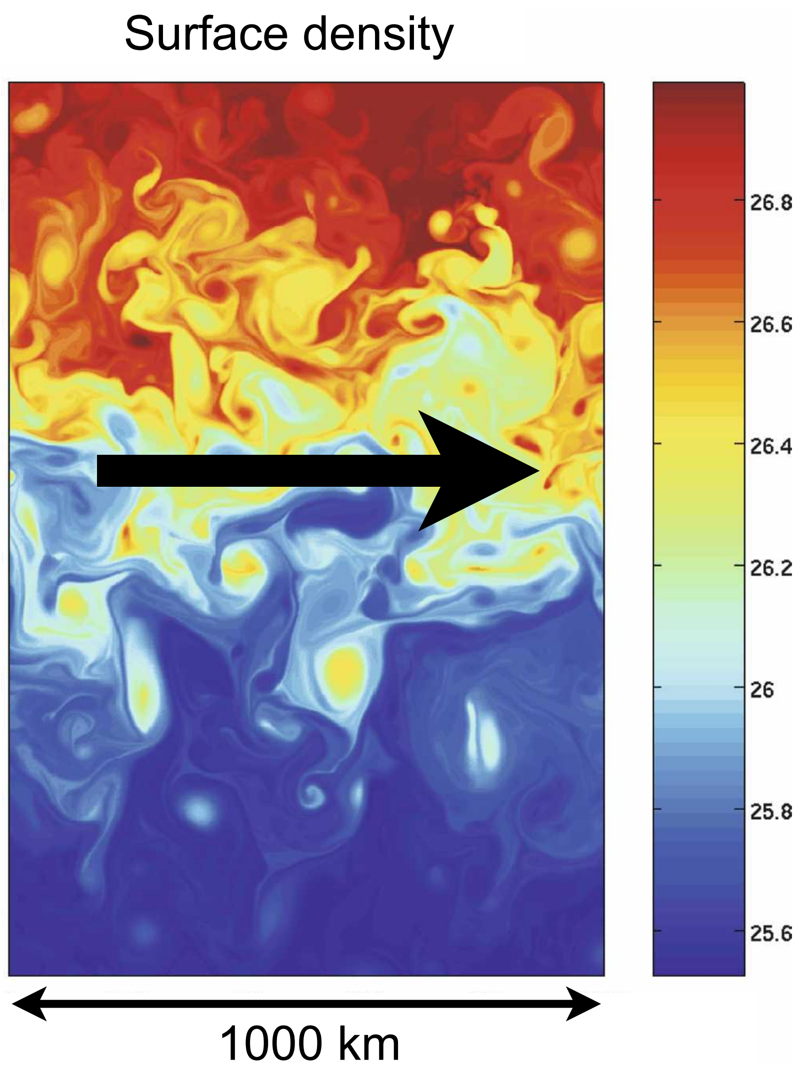
$$\psi(k,z) = \frac{g}{N_0 \rho_0 k}(-\hat{\rho_s}) \exp\left[\frac{N_0 k}{f_0} z\right]$$

Horizontal streamfunction



# Imposing a mixed layer

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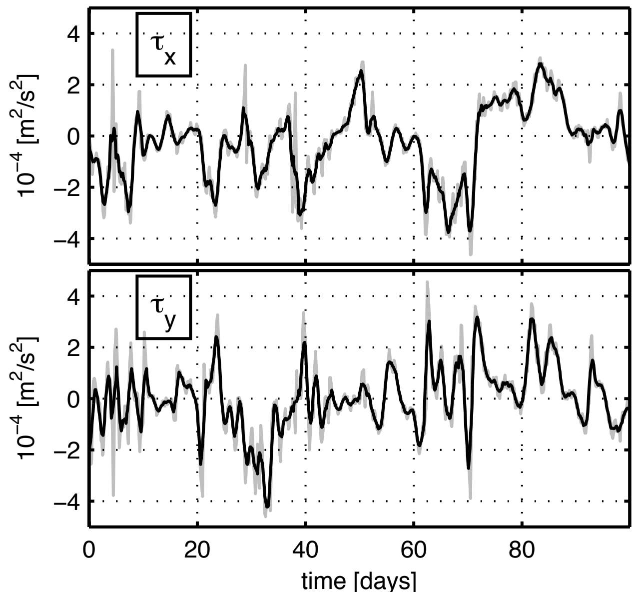
**Wind stress**
**Spatially uniform**
**Time varying**



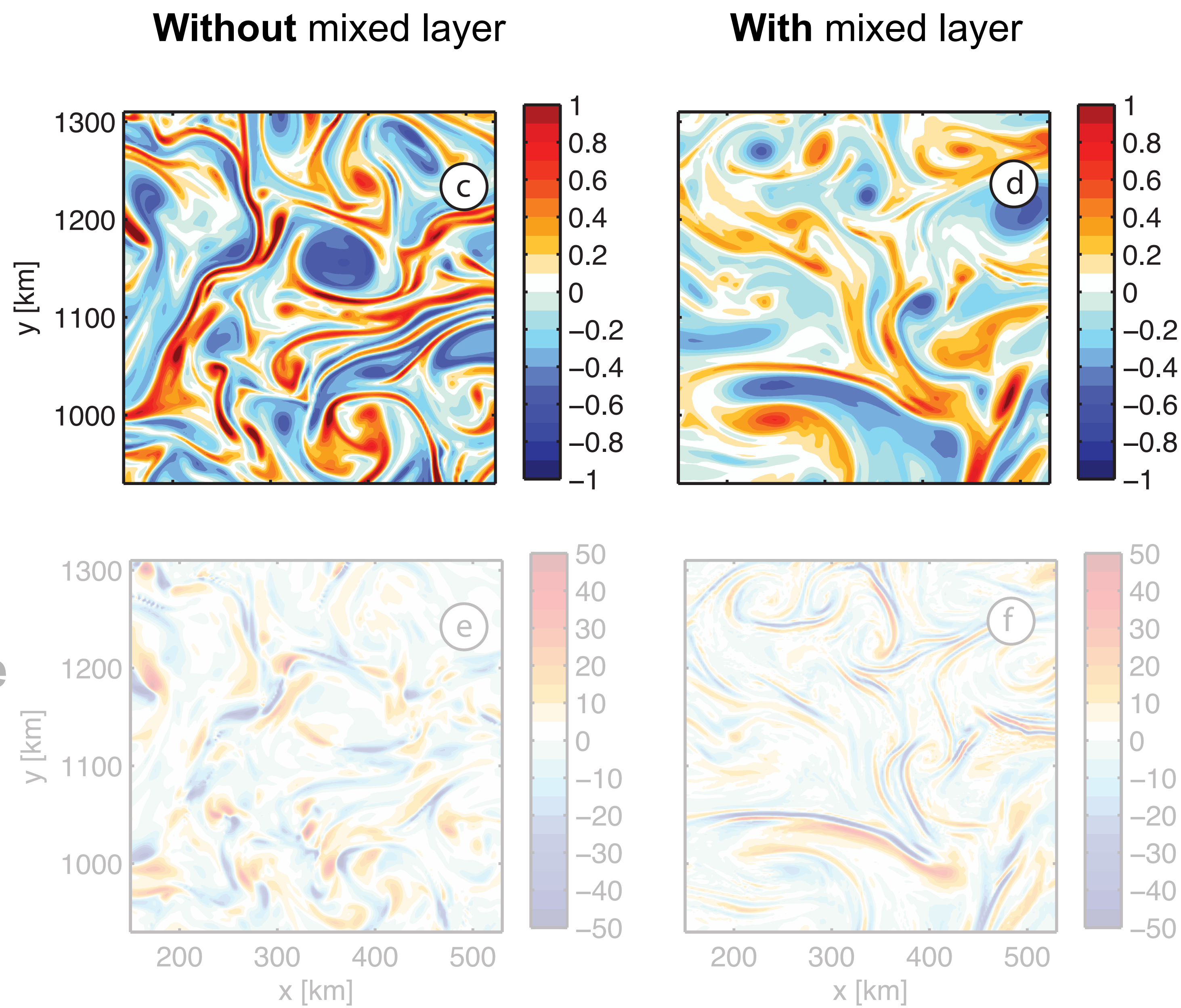

**Mixed layer depth ~ 60-70m**

**Comparison between simulations with and without mixed layer**

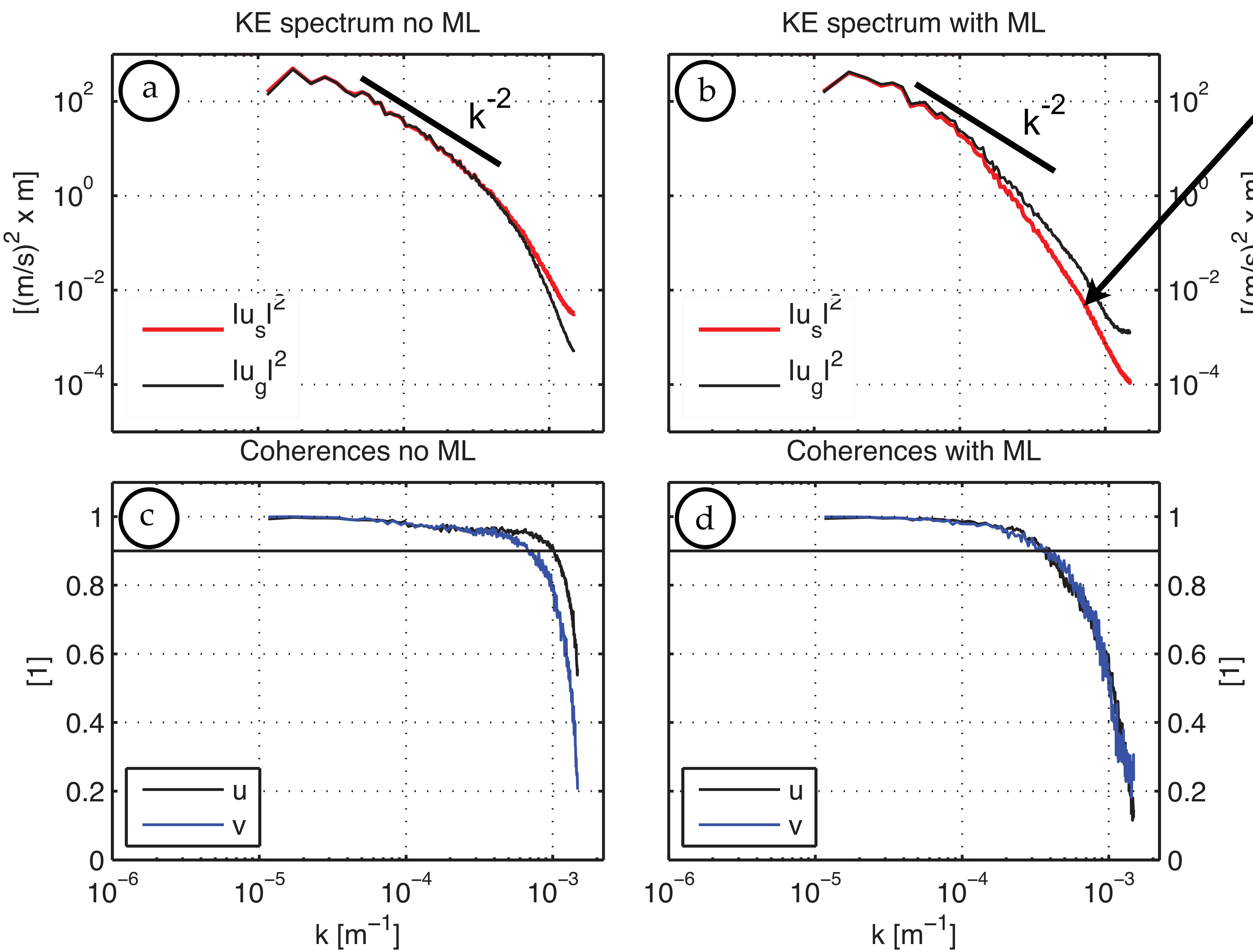
# Visual differences

**relative vorticity**
**weaker vorticity**
**Ro~0.6 without ML**
**Ro~0.3 with ML**

**w(z=-40m)**
**filamentary structure of w astride density fronts with ML**



# Horizontal motions

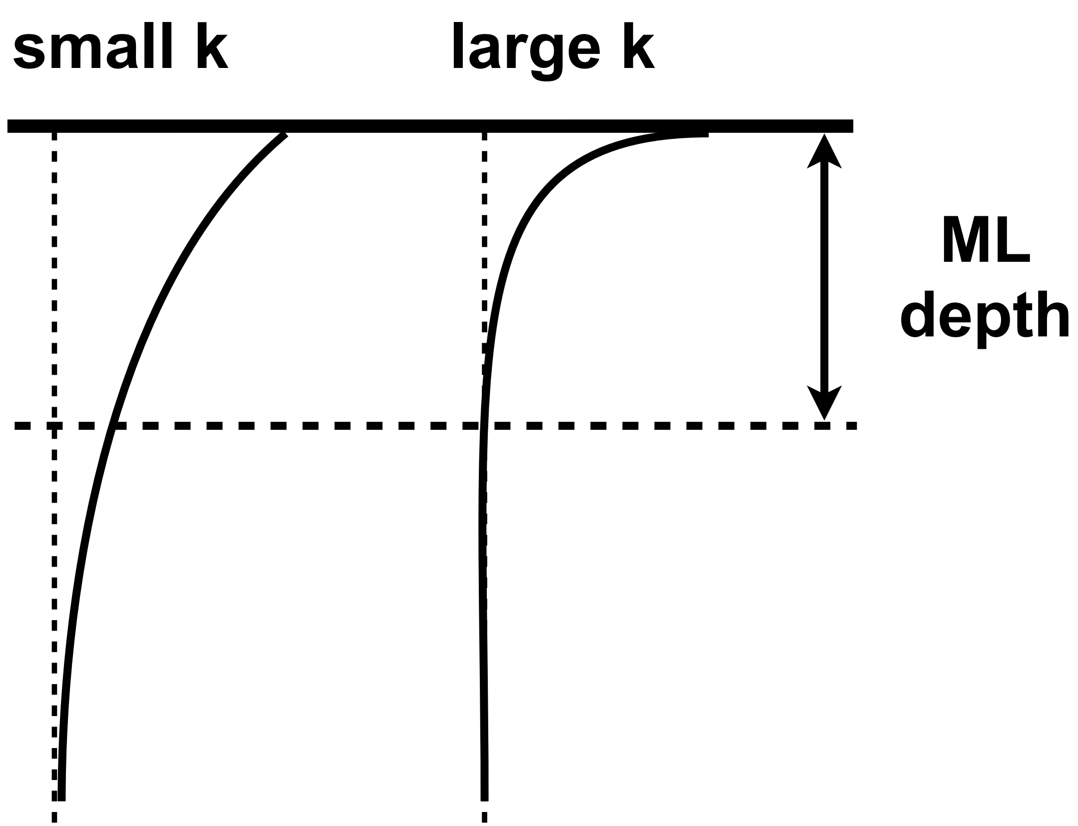



**steeper**
**+**
**larger ageostrophy**

# Horizontal motions: heuristic explanation

$$\widehat{\psi}(\mathbf{k}, z) = \frac{g}{f}\widehat{\eta}(\mathbf{k}) \exp\left(\frac{N_0}{f}kz\right)$$

**eSQG solution (SQG dynamics)**

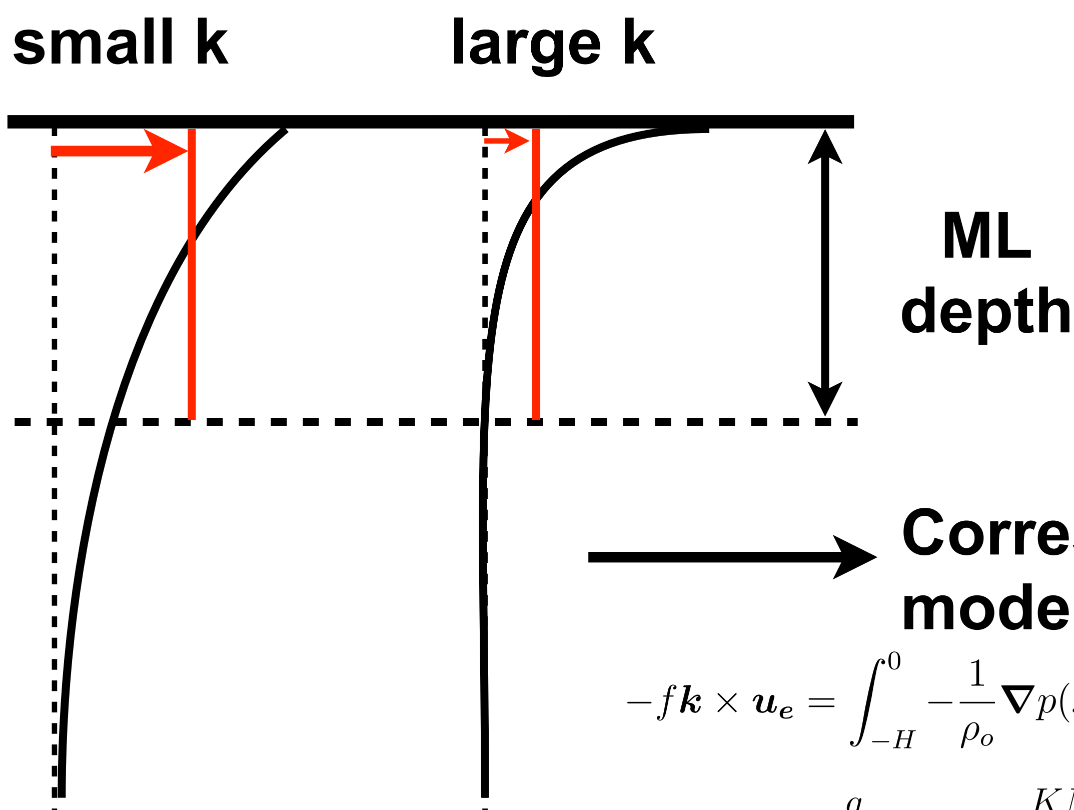


# Horizontal motions: heuristic model

$$\widehat{\psi}(\mathbf{k},z) = \frac{g}{f}\widehat{\eta}(\mathbf{k})\exp\left(\frac{N_0}{f}kz\right)$$

**eSQG solution (SQG dynamics)**

**Mixing smoothes profiles**




**Corresponding model:**

$$-f\boldsymbol{k}\times\boldsymbol{u_e} = \int_{-H}^{0} -\frac{1}{\rho_o}\boldsymbol{\nabla}p(x,y,z)dz/H + \frac{\boldsymbol{\tau}}{\rho_o H}\,,$$

$$\widehat{p}(\mathbf{K},z) = \frac{g}{f}\widehat{\eta}(\mathbf{K})\exp(\frac{KN_e}{f}z) \quad \textbf{(eSQG)}$$

$$\widehat{\boldsymbol{u_e}}(K_x,K_y) = \widehat{\boldsymbol{u_g}}(K_x,K_y,0)\frac{f}{KN_eH}[1-exp(\frac{-KN_eH}{f})]\,,$$

# Horizontal motions: heuristic model

$$\widehat{\boldsymbol{u_e}}(K_x, K_y) = \widehat{\boldsymbol{u_g}}(K_x, K_y, 0)\frac{f}{KN_eH}[1 - exp(\frac{-KN_eH}{f})] \ ,$$

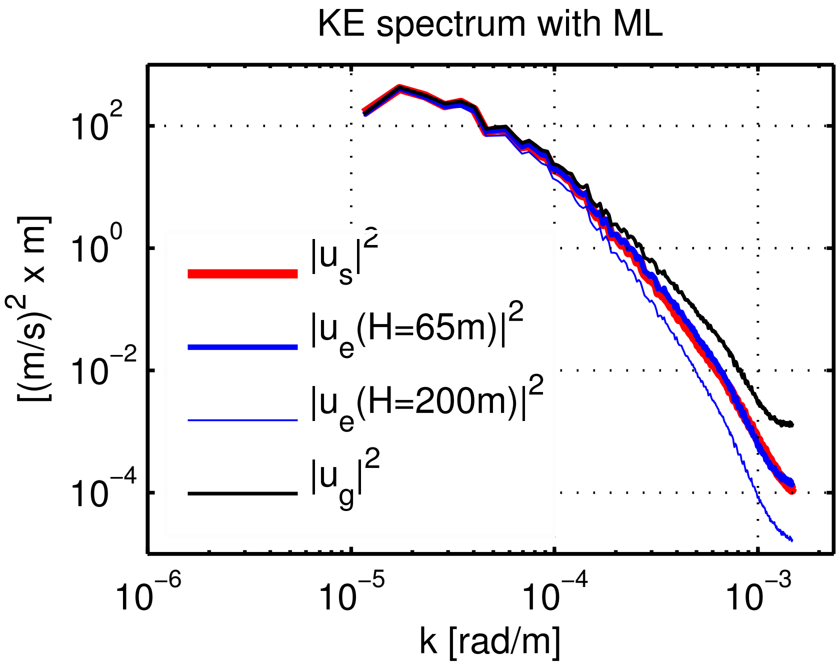
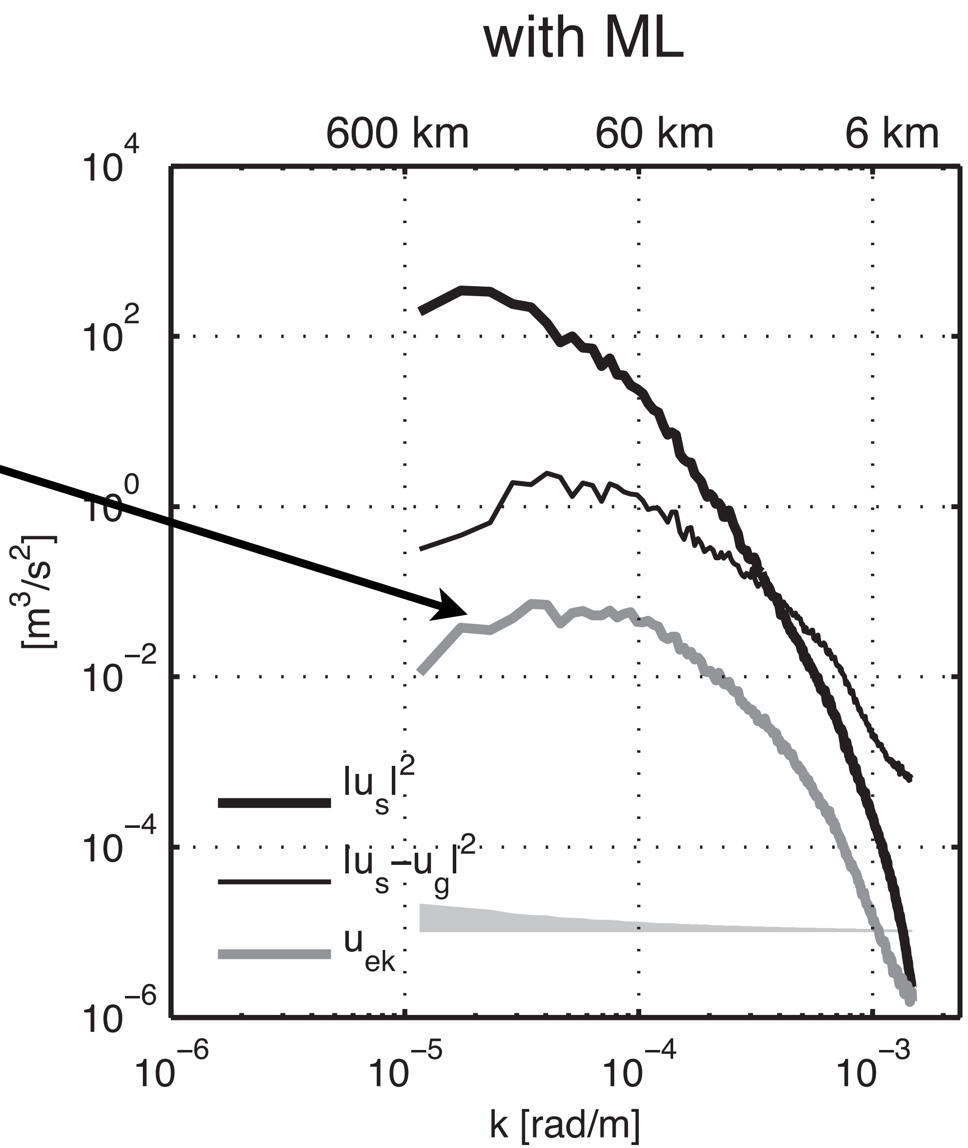



FIG. 4. Surface velocity spectrum estimated from the SSH using geostrophy (black curve), from the velocity observed at the surface (red curve) and from (4) and $\boldsymbol{u_g}$ (using $N_e/f = 30$) (blue curve) in the simulation with a 65m deep ML. Units on the vertical axis are in $m^3s^{-2}$. $k_h = 10^{-4}$ rad/m corresponds to a wavelength of 60km. The thin blue curve corresponds to a surface velocity spectrum estimated from (4) for a ML depth of 200m.

# Horizontal motions: Ekman response ?

**Nonlinear Ekman response (Stern 1965, Niiler 1969)**

$$u = \frac{\tau_y/\rho_0}{H_{ml}(f+\xi)}$$




**Frozen vorticity field -> small**

**+**

**Issue of time dependence of the wind and of submesoscale structures**

# Visual differences

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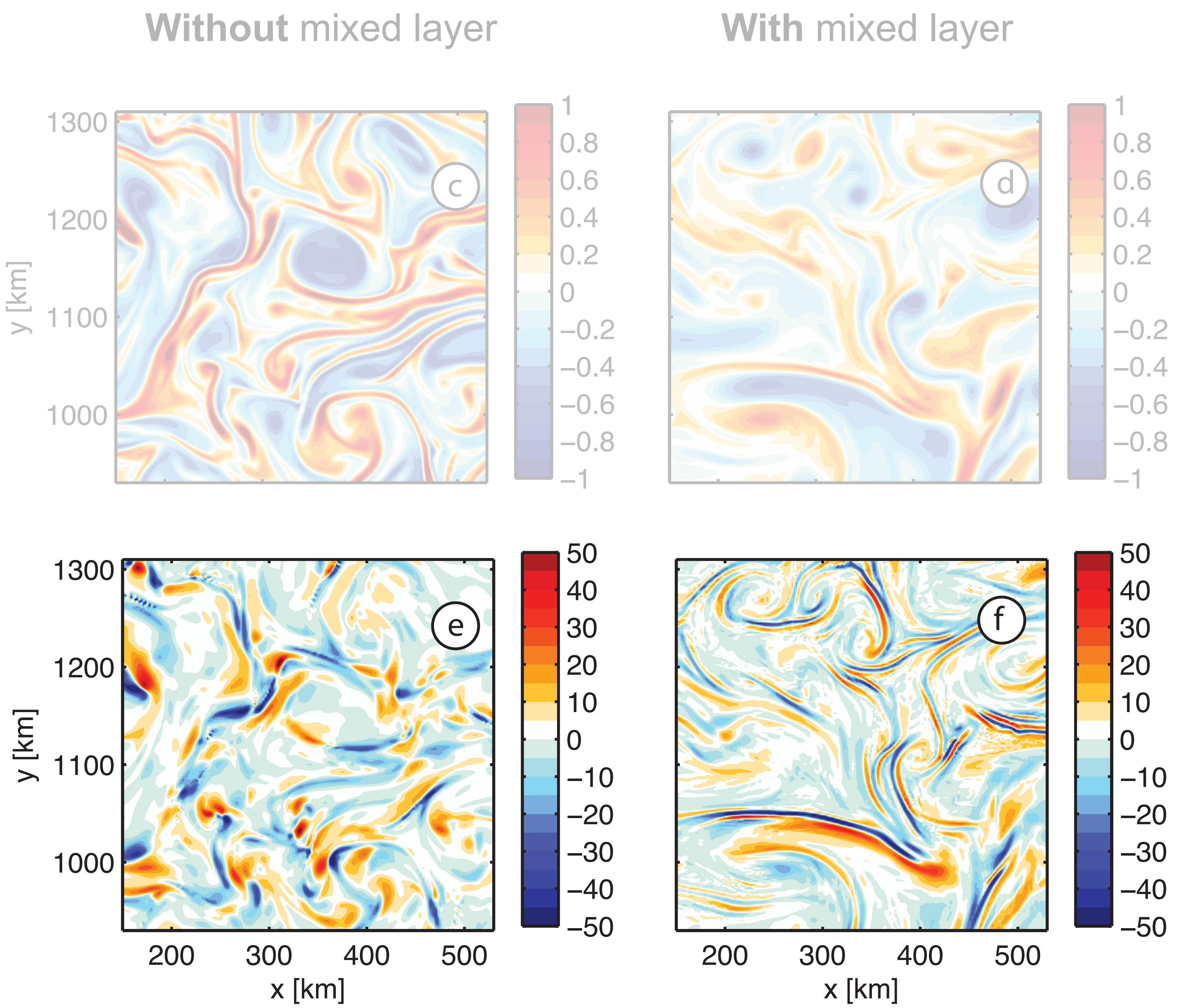
weaker vorticity
Ro~0.6 without ML
Ro~0.3 with ML

**w(z=-40m)**
**filamentary structure of w astride density fronts with ML**



# Recontruction of the vertical velocity

QG Omega equation:

$$N^2 \Delta w + f^2 \frac{\partial^2 w}{\partial z^2} = \boldsymbol{\nabla} \cdot \mathbf{Q_{kd}} \quad ,$$

$$\mathbf{Q_{kd}} = (\nabla \mathbf{u_h})^t \cdot \nabla \rho .$$

SQG vertical velocity

$$\widehat{w}(\mathbf{k}, z) = -\frac{c^2}{N_0^2} \left[ -\widehat{J(\psi_s, b_s)} \exp\left(\frac{N_0}{f} kz\right) + \widehat{J(\psi, b)} \right]$$

*(Klein et al. 2009)*

+

Garrett and Loder 1981

$$w_m \approx \frac{g}{f^2 \rho_o} A_v \Delta \rho \; .$$

$$-fv = A_v \partial_{zz} u$$

$$-fv = \frac{g A_v}{f \rho_0} \partial_z \partial_y \rho$$

$$-f \partial_y v = \frac{g A_v}{f \rho_0} \partial_z \partial_{yy} \rho$$

...

Thermal wind

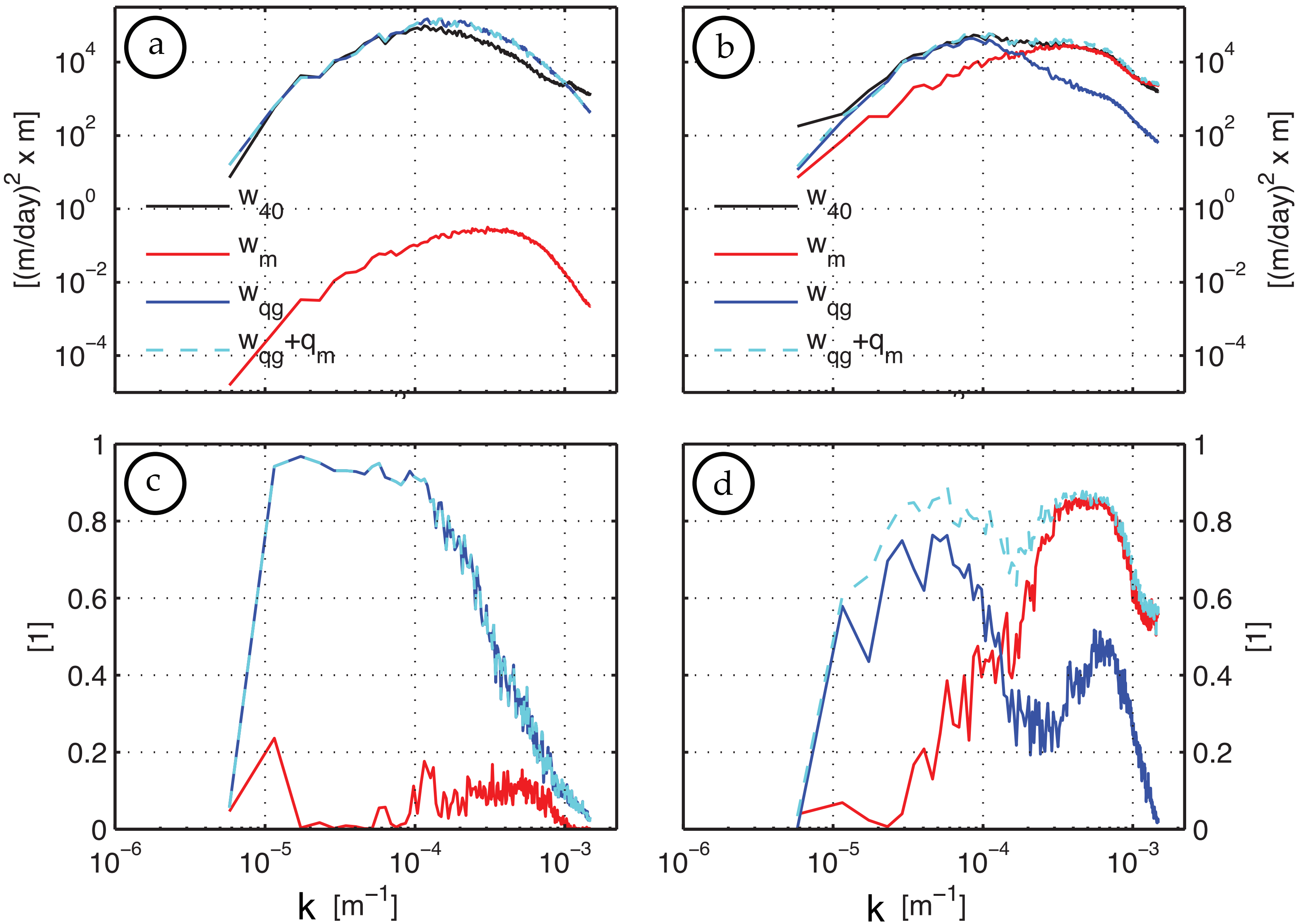
$\partial_y$

→ Turbulent buoyancy forcing term
Generalized Omega equation
Giordani et al. 2005

# Vertical motions

**Without** mixed layer

**With** mixed layer




**Spectra**

**Coherence**

# Vertical motions: physical space

## With mixed layer

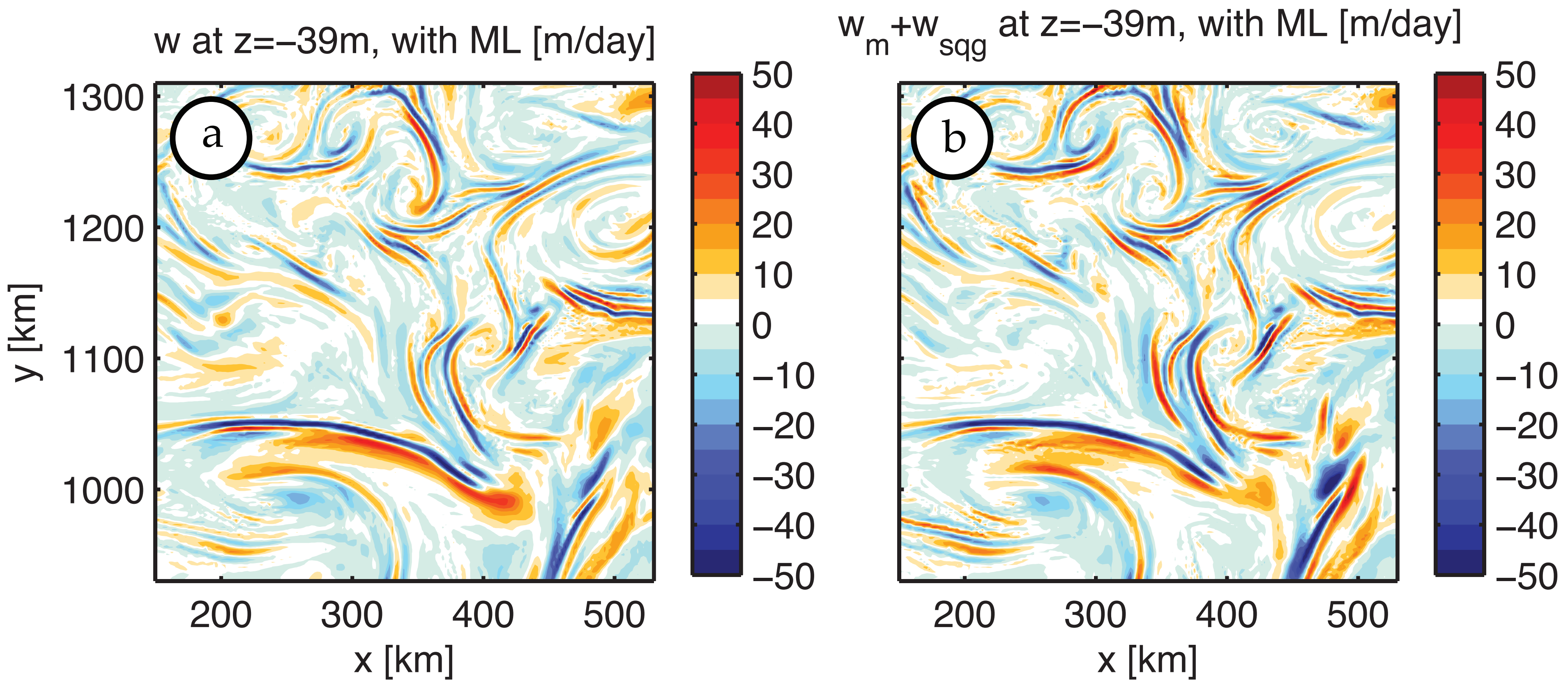



FIG. 6. Snapshots of the vertical velocity field at 40 m with ML (a) and reconstructed field $w_{sqg} + w_m$ (b). Units are m/day.

# Conclusion

**Vertical mixing was shown:**
**- to decrease the magnitude of horizontal motions**
**- increase the magnitude of small scale vertical motions**

**It is possible to account for the effect of vertical mixing on horizontal motions with a simple analytical model provided current at the surface is known and mixed layer depth**

$$\widehat{\boldsymbol{u_e}}(K_x, K_y) = \widehat{\boldsymbol{u_g}}(K_x, K_y, 0)\frac{f}{KN_eH}[1 - exp(\frac{-KN_eH}{f})] \ ,$$

**It is possible to reconstruct the vertical velocity field from 3D density and horizontal currents the vertical velocity field as well as vertical mixing.**

$$w_m \approx \frac{g}{f^2\rho_o}A_v\Delta\rho \ .$$

*Ponte et al. 2013*