Stochastic super-resolution of satellite imagery of the ocean

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2D to 3D Ocean Dynamics from Space
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Fronts and eddies on O(10) km play a critical but poorly understood role in:

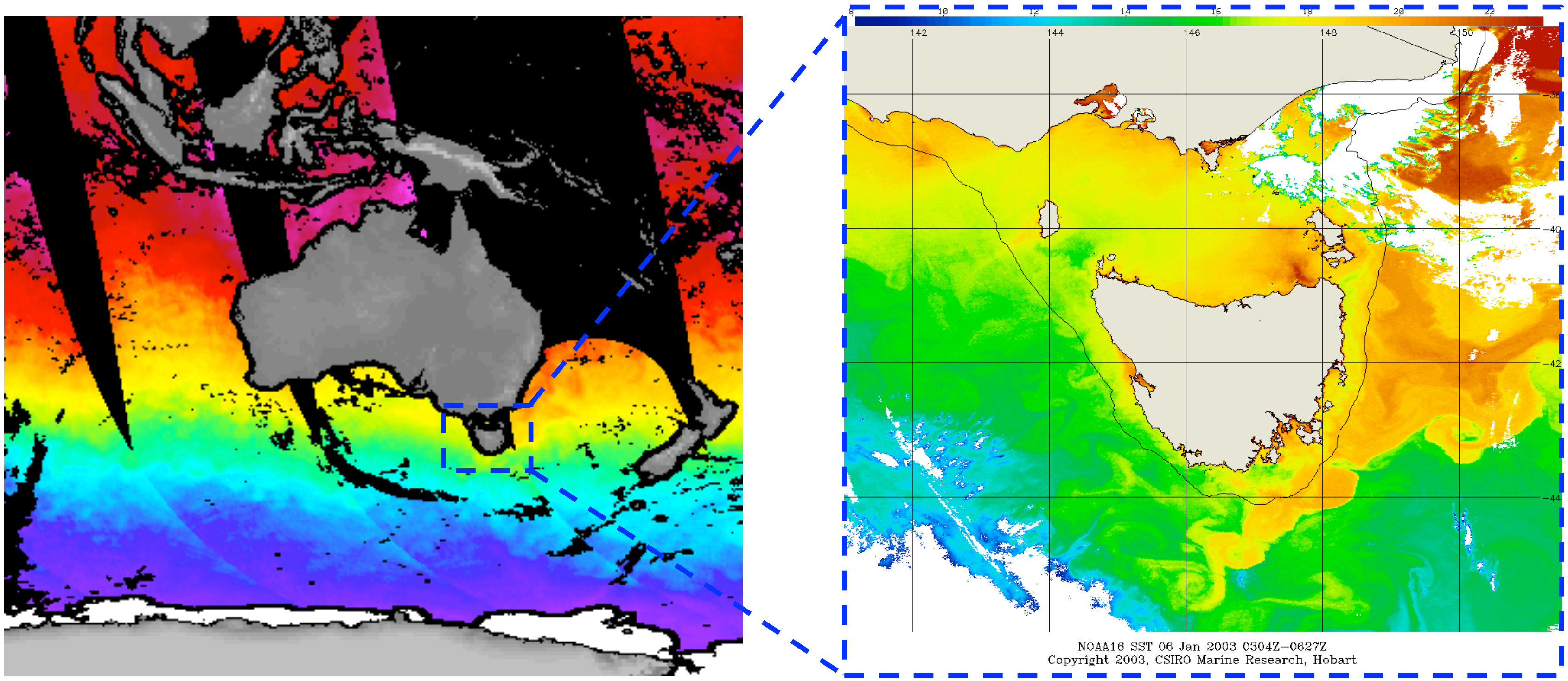
- *vertical and horizontal mixing*
- *global transport and uptake*
- *biogeochemical interactions*

Observation of the upper ocean on these scales is a **key challenge.**

A need for **new analytical tools** for probing these scales.

Ocean color / Chlorophyll
Credit: European Space Agency

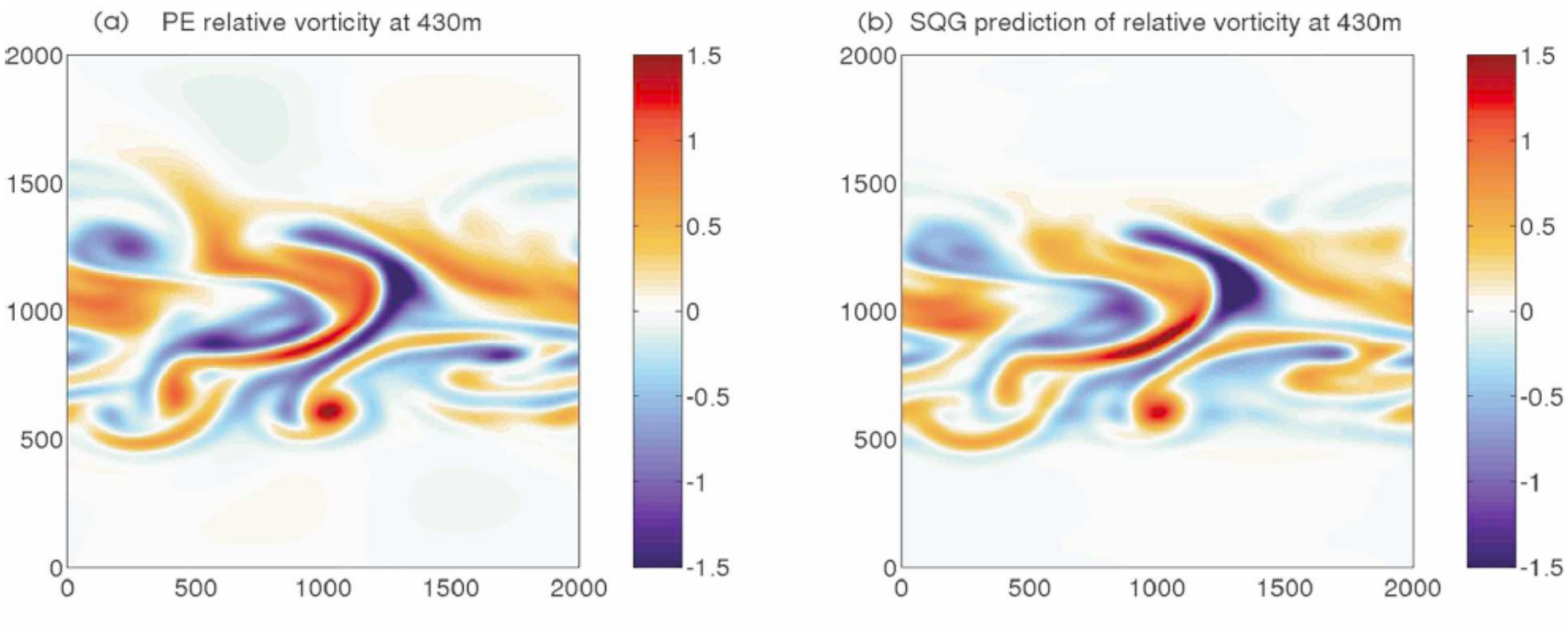
Microwave SST (AMSR-E) | Infrared SST (AVHRR)



SST observations preserve information about the flow:

- **Microwave observations** have spatial resolutions of **20-50 km** and can **penetrate clouds**
- **Infrared observations** have spatial resolutions of **1 km** but are **obscured by clouds**

Dynamical reconstruction of subsurface flow (Lapeyre & Klein 2006)



Lapeyre & Klein (2006)

- **Surface quasigeostrophic (SQG) model**: Interior streamfunction slaved to surface density (temperature) anomalies.

$$\hat{\psi}_{\mathrm{sqg}}(\mathbf{k},z) \sim \frac{\hat{\theta}_{\mathrm{s}}(\mathbf{k})}{K}\exp\left(\frac{NKz}{f}\right)$$

- Streamfunction is **smoothed version** of temperature: Microwave observations reconstructs flow with resolution of O(100) km.

- Derive **super-resolved SST images** by combining microwave observations with **statistical knowledge** from infrared images
- Exploit **spatial aliasing** of small scales by **coarse observations**

Original image

Subsampled image

Aliasing of sparse observations

Fourier transform on **coarse** (*M x M*) grid:

$$\psi_{k,l}^{coarse} = \frac{1}{M^2} \sum_{m,n=1}^{M} \psi\ (mH, nH)\, e^{iH(mk+nl)}$$

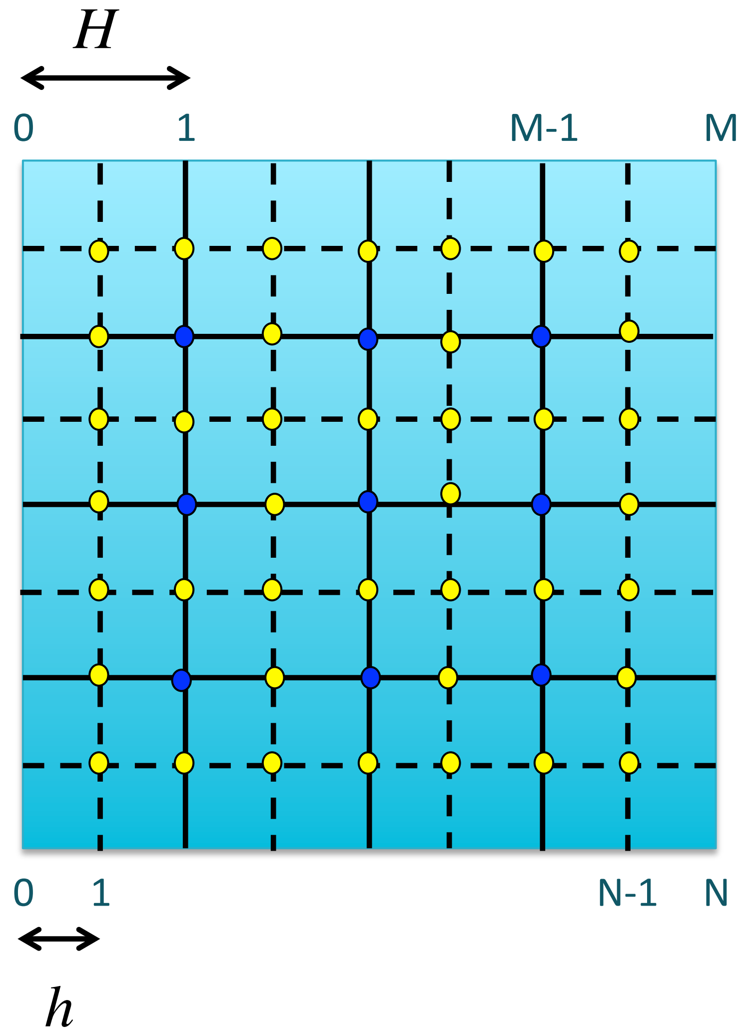
Fourier transform on **fine** (*N x N*) grid:

$$\psi_{\tilde{k},\tilde{l}}^{fine} = \frac{1}{N^2} \sum_{m,n=1}^{N} \psi\ (mh, nh)\, e^{ih(m\tilde{k}+n\tilde{l})}$$

Coarse-grid modes are superposition of fine-grid modes in **same aliasing set.**

$$\psi_{k,l}^{coarse} = \sum_{\tilde{k},\tilde{l}} \psi_{\tilde{k},\tilde{l}}^{fine} \qquad \begin{aligned} \tilde{k} \bmod \mathrm{M} &= k \\ \tilde{l} \bmod \mathrm{M} &= l \end{aligned}$$

Reconstruct **super-resolved image** by combining observations and prediction.



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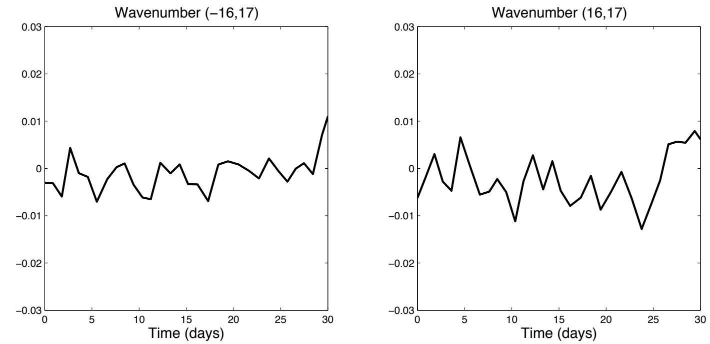

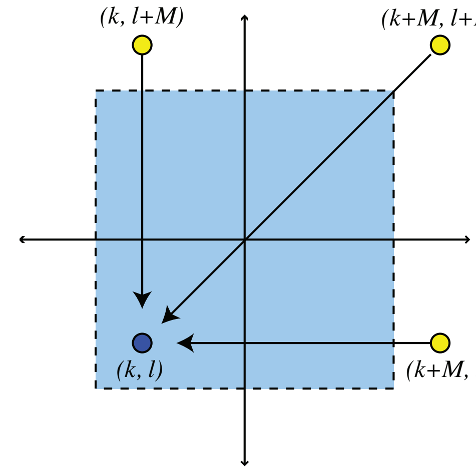

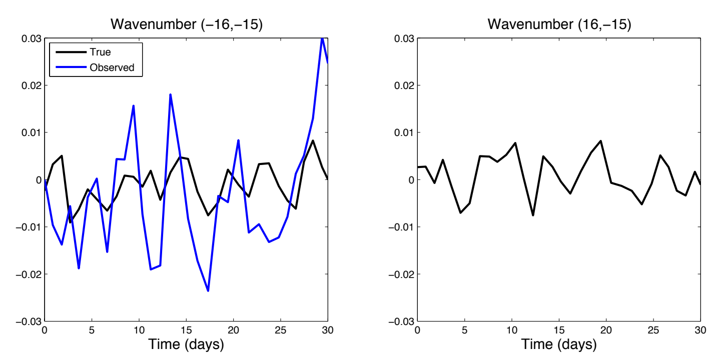

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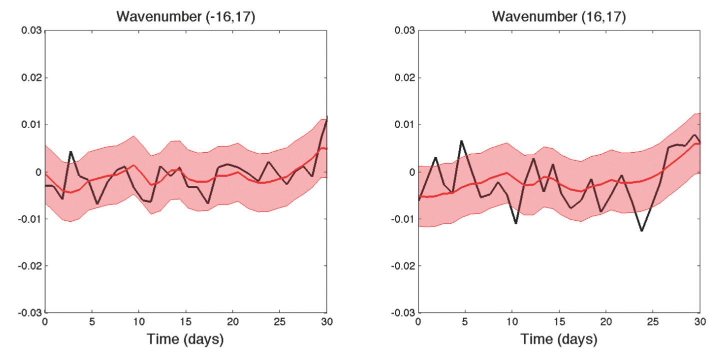
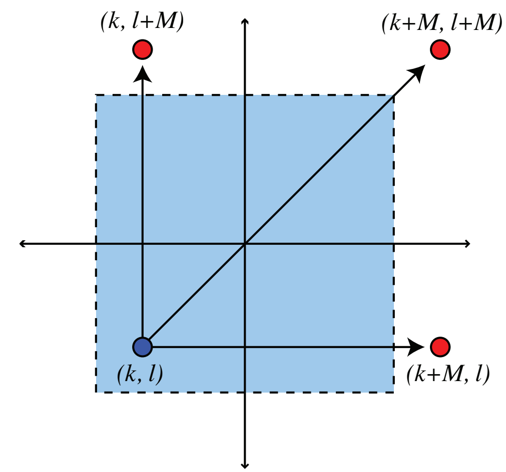
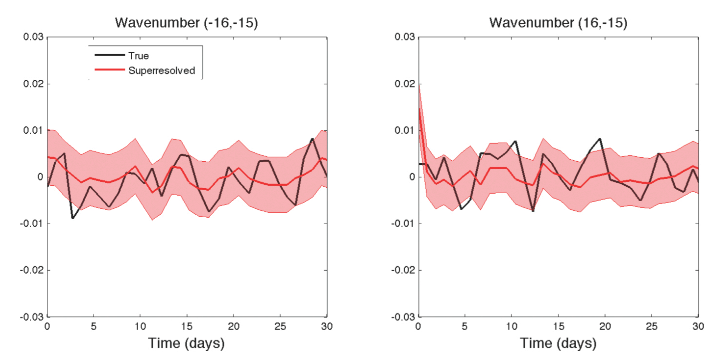
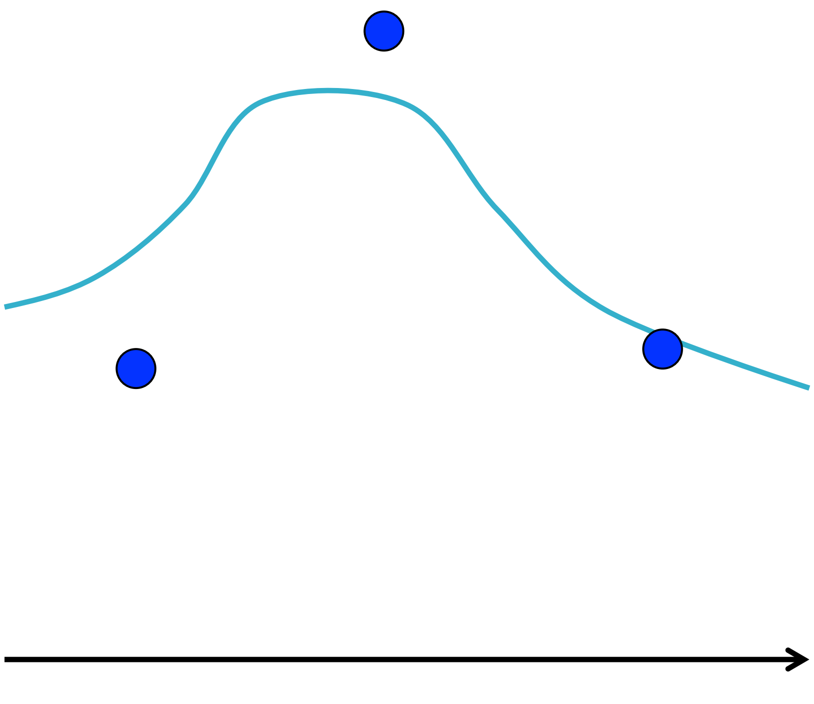






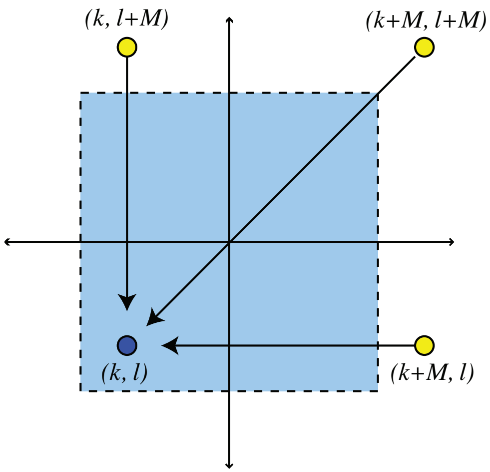
Filtering sparse observations

Data assimilation or filtering seeks the *best-guess estimate* of the *state of the system* by combining *noisy, incomplete observations* with an *internal forecast model.*



M x M observations of each resolved mode + aliased modes

$$\psi_{k,l}^{obs} = \sum_{\tilde{k},\tilde{l}} \psi_{\tilde{k},\tilde{l}}^{true} + \sigma_{k,l}^{obs}$$



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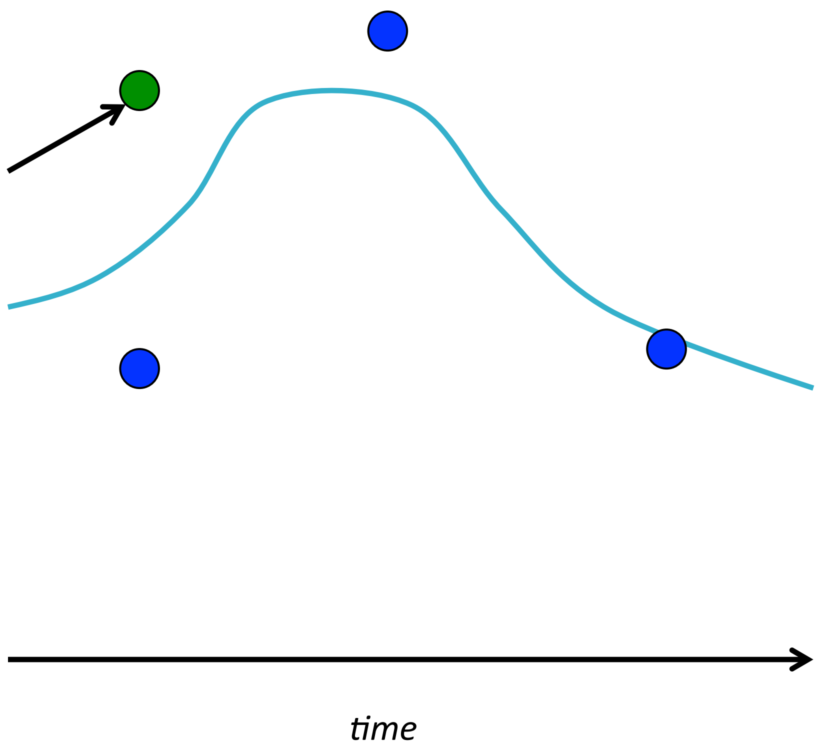

1. Forecast step:

Make prediction for *N x N* modes using quasi-linear stochastic model.

$$\partial_t \hat{\theta} = -(\gamma - \mathrm{i}\omega)\hat{\theta}(t) + \sigma \dot{W}(t)$$

Forecast mean and covariance:

$$\langle \theta \rangle, \quad R_{pq} = \langle \theta_p^* \theta_q \rangle$$

Tune parameters to give correct energy and timescales estimated from **infrared observations**.

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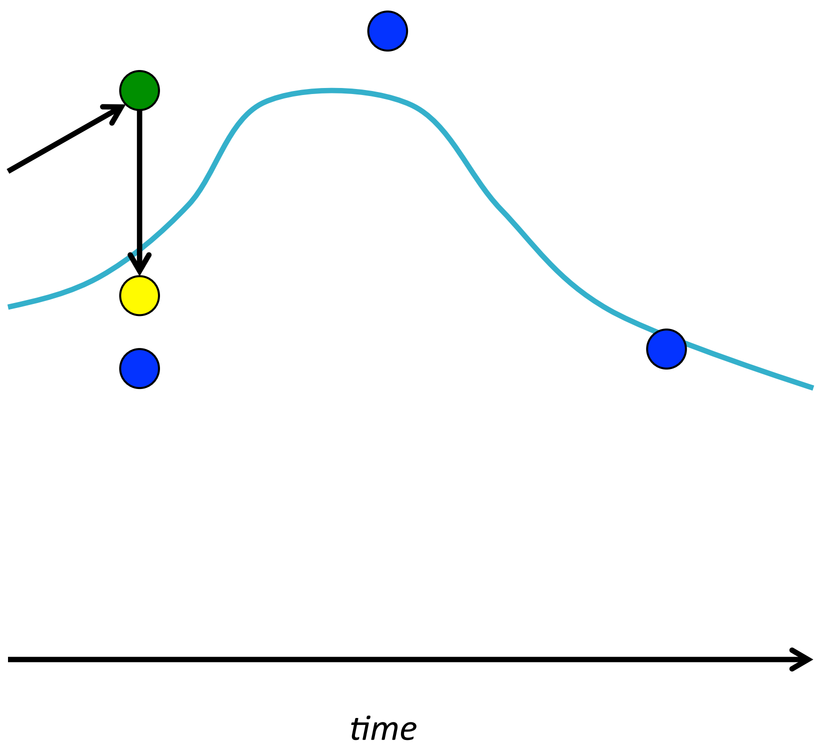

2. Update step:

Combine *N x N* prediction (-) with *M x M* observation (~) using **Kalman filter** solution:

$$\langle \theta_+ \rangle = (1 - KG)\langle \theta_- \rangle + K\tilde{\theta}$$

$$R_+ = (1 - KG) R_-$$

Optimal solution when dynamics and observation operator are linear with unbiased uncorrelated Gaussian noise.

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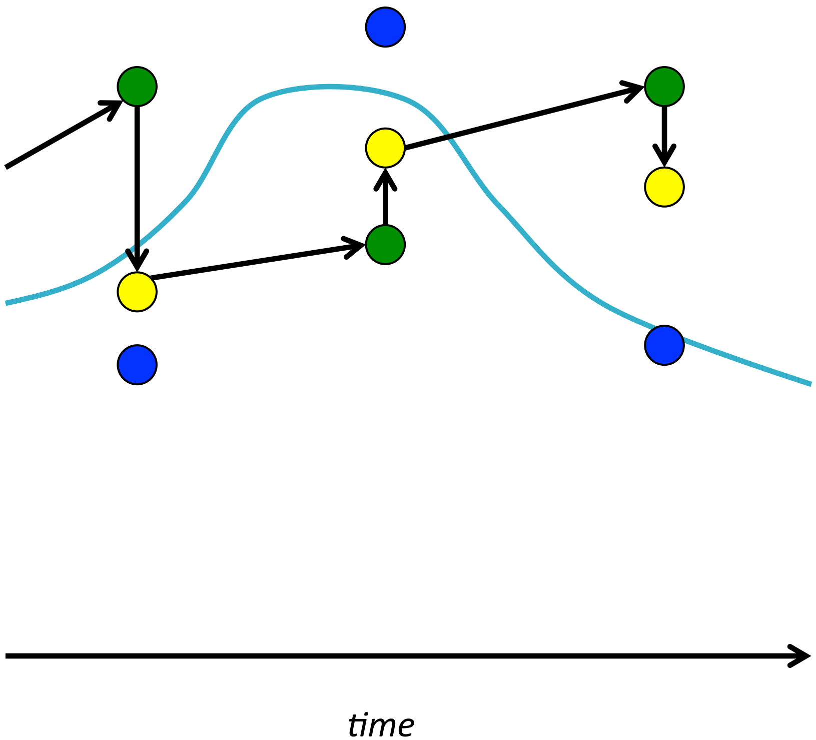

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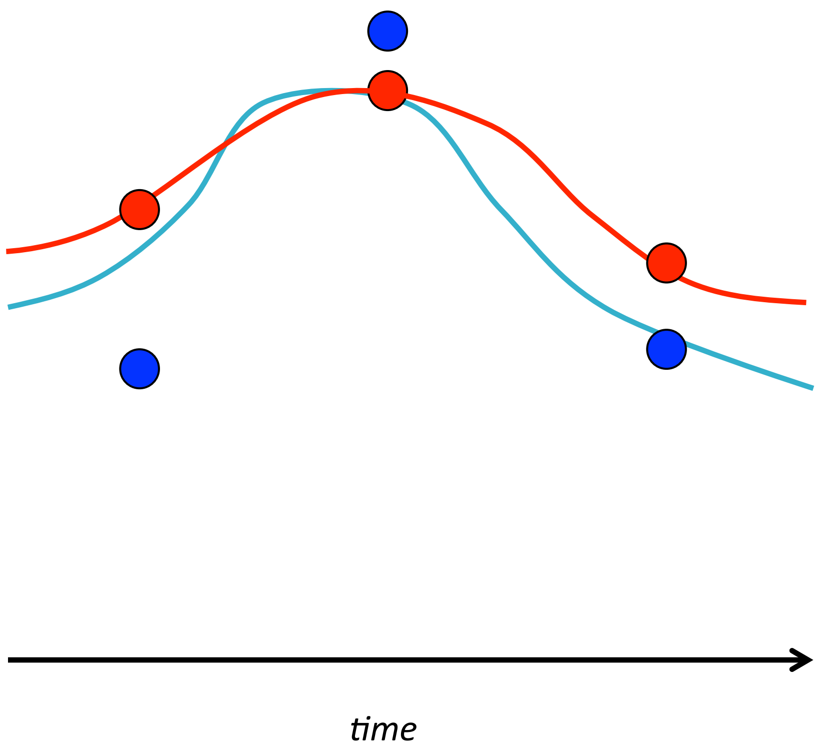

3. Smoothing step:

Apply **RTS smoother** after Kalman filter to remove unphysical jumps in the temperature field.

Resulting **super-resolved SST estimate** is a statistical distribution with given mean and covariance.

Effective resolution given by *N x N* forecast model, rather than *M x M* observations.

Filtering sparse observations

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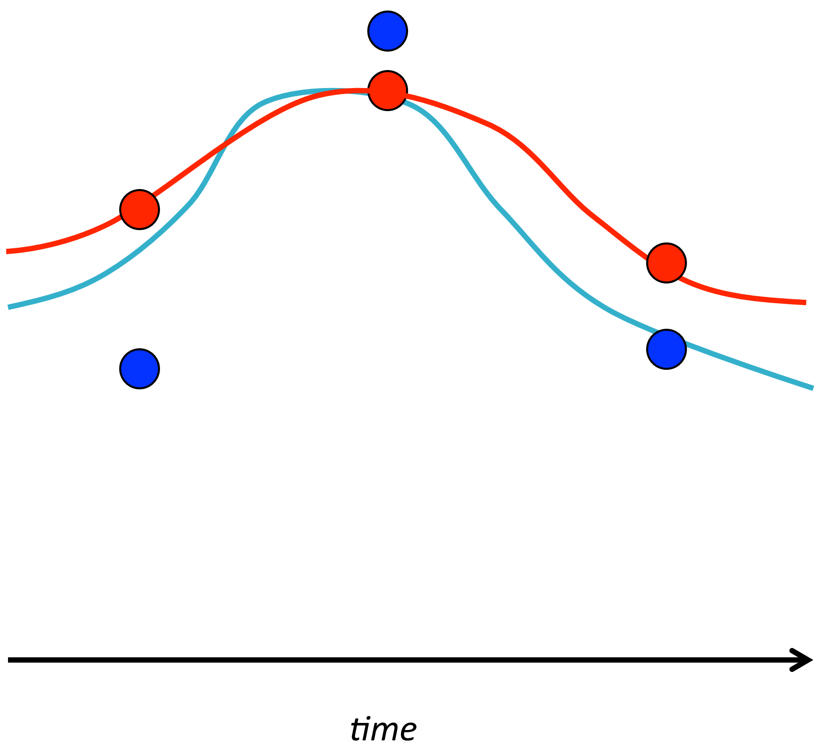
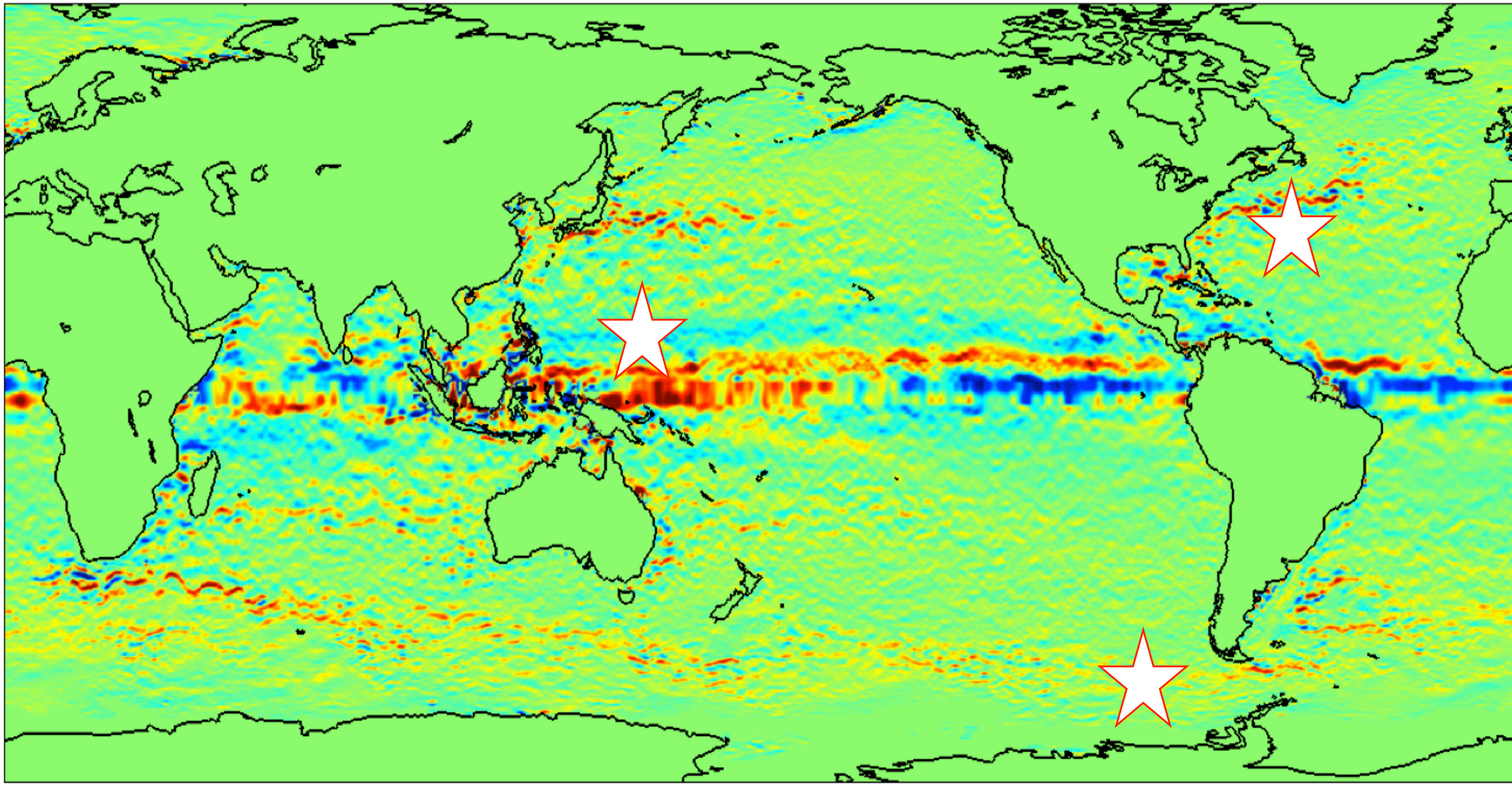


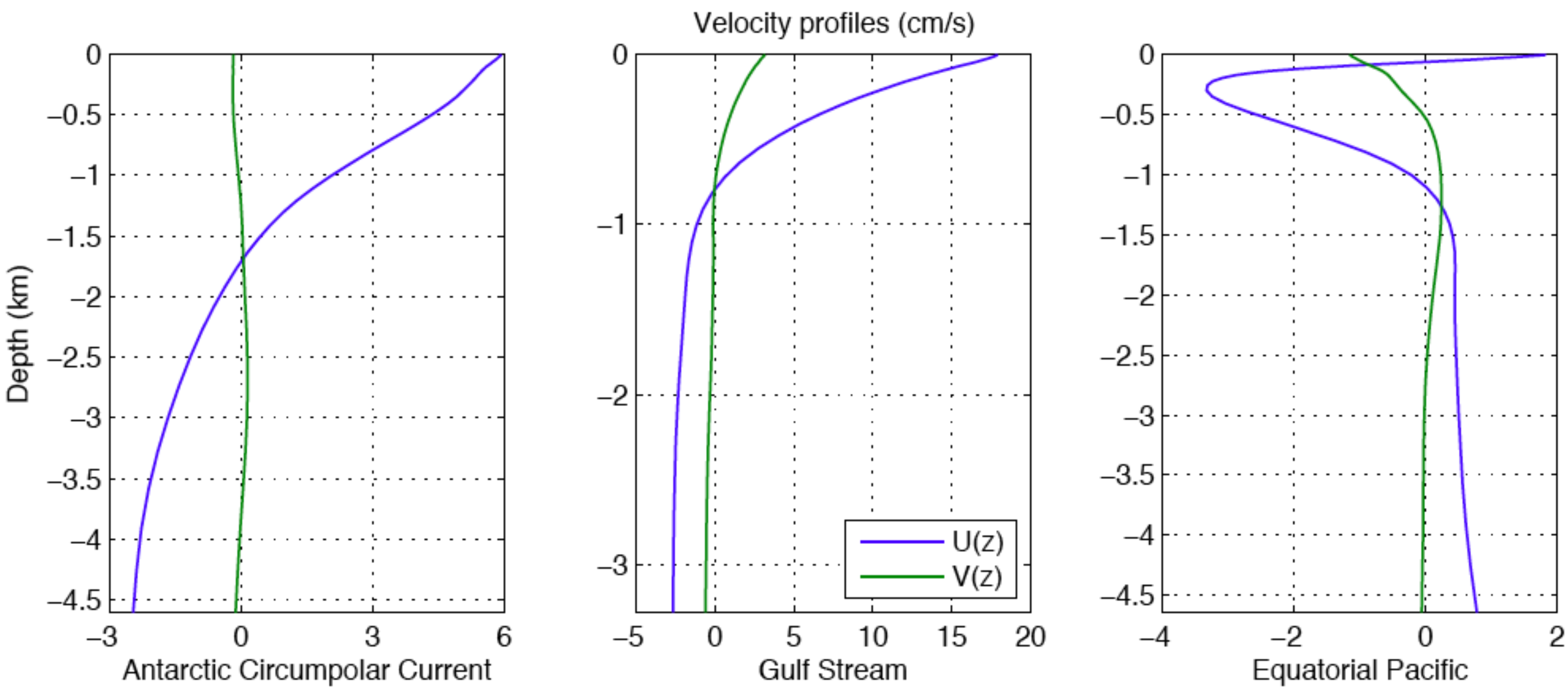
Filtering is *not* the same as **projection** (onto EOF basis, BC/BT modes etc).

It is a **statistical inference** about the *full* system **constrained** by observations.

Information about **unobserved** variables obtained because the evolution operator induces correlations with **observed** variables.



- Test in **QG simulations** driven by Forget (2010) hydrography.
- Assume that **surface density anomalies** are dominated by SST.
- Synthetic daily temperature observations over a 90-day period with both **microwave (40 km) and infrared (5 km) resolutions**.
- Infrared observations used to learn **stochastic parameters**.



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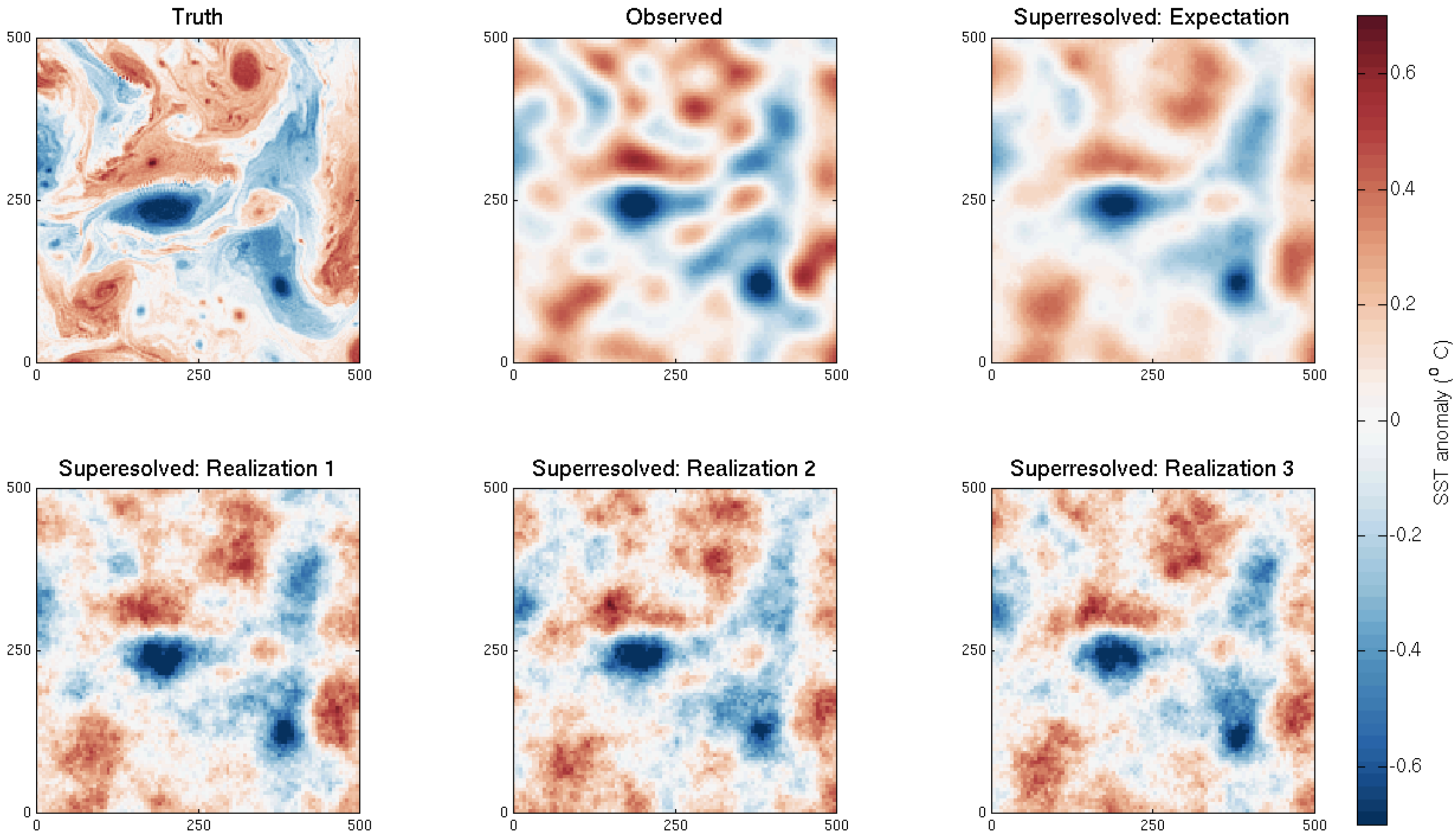
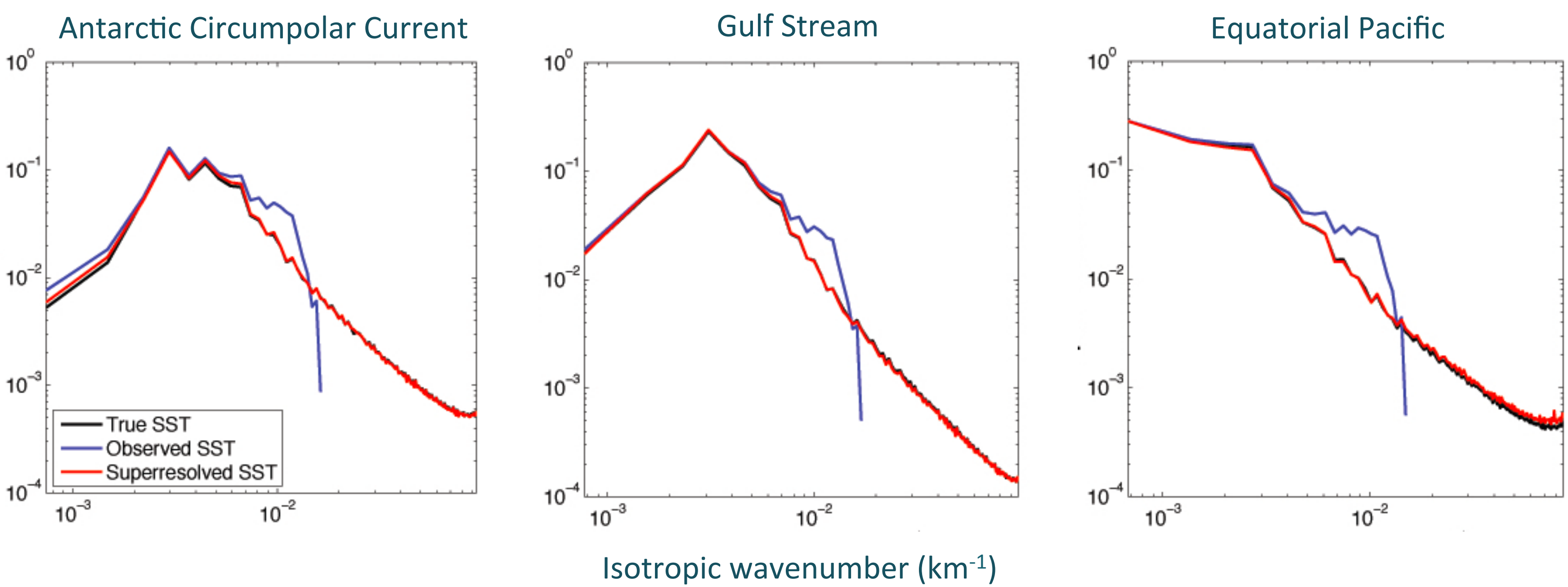
SST snapshots: Antarctic Circumpolar Current



$$\theta_{kl} = \langle \theta_{kl} \rangle + A(k,l)X, \qquad A^*(k,l)A(k,l) = R(k,l)$$

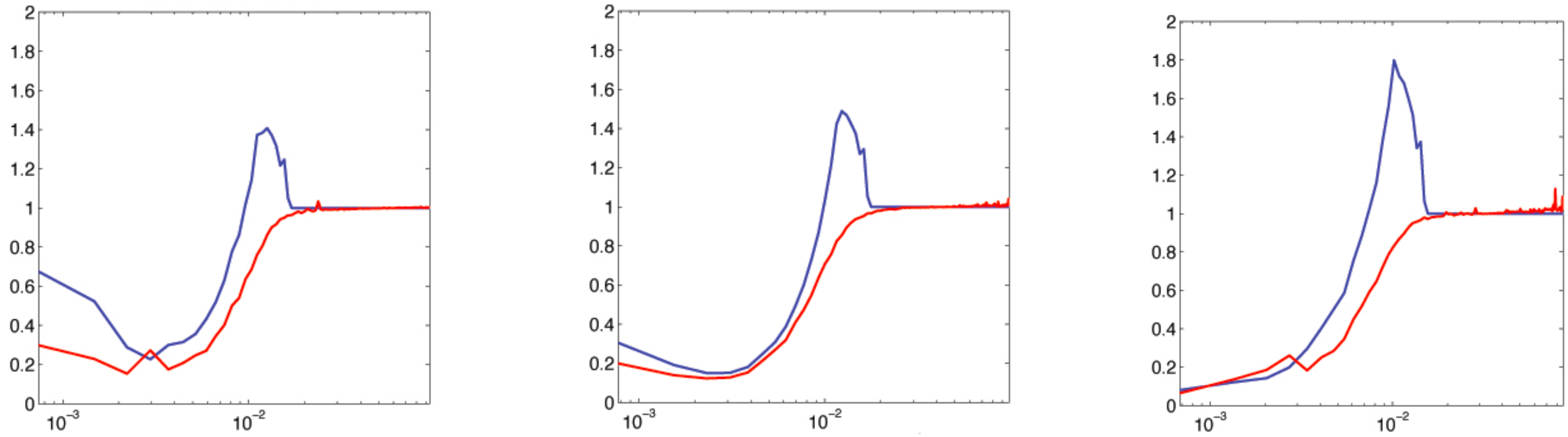
Temperature variance spectrum: $\left\langle \left|\theta(k)\right|^2 \right\rangle$



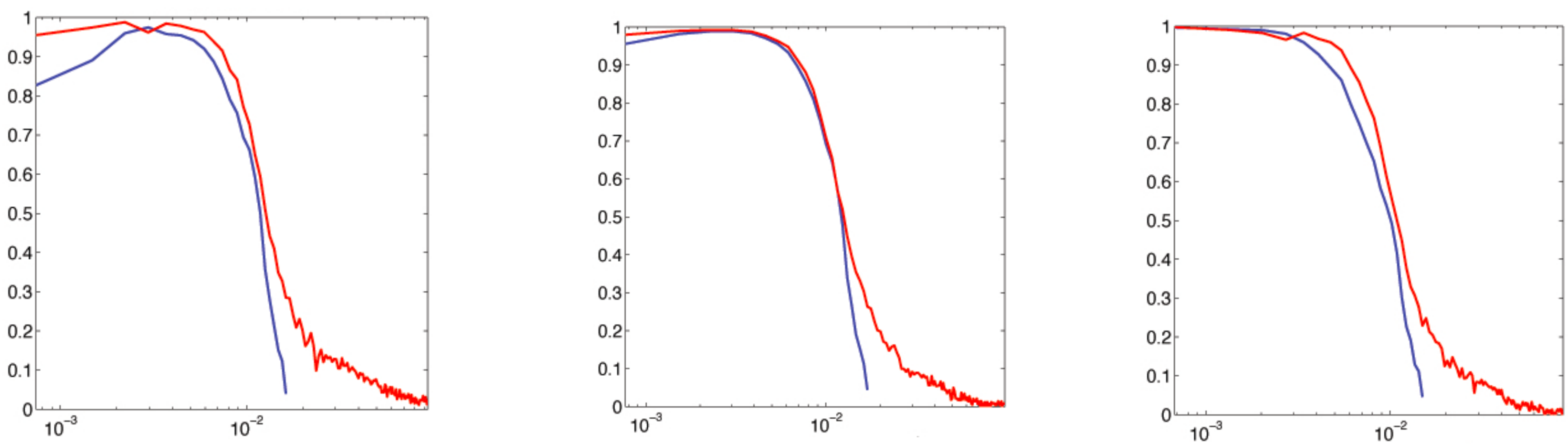
- **Effect of aliasing** can be seen in spurious variance in observations near the limit of resolution
- Super-resolved estimate correctly **redistributes variance** to small scales

RMS error: $\left\langle \left|\theta(k)-\theta^{true}(k)\right|^2 \right\rangle^{1/2} \Big/ \left\langle \left|\theta^{true}(k)\right|^2 \right\rangle^{1/2}$

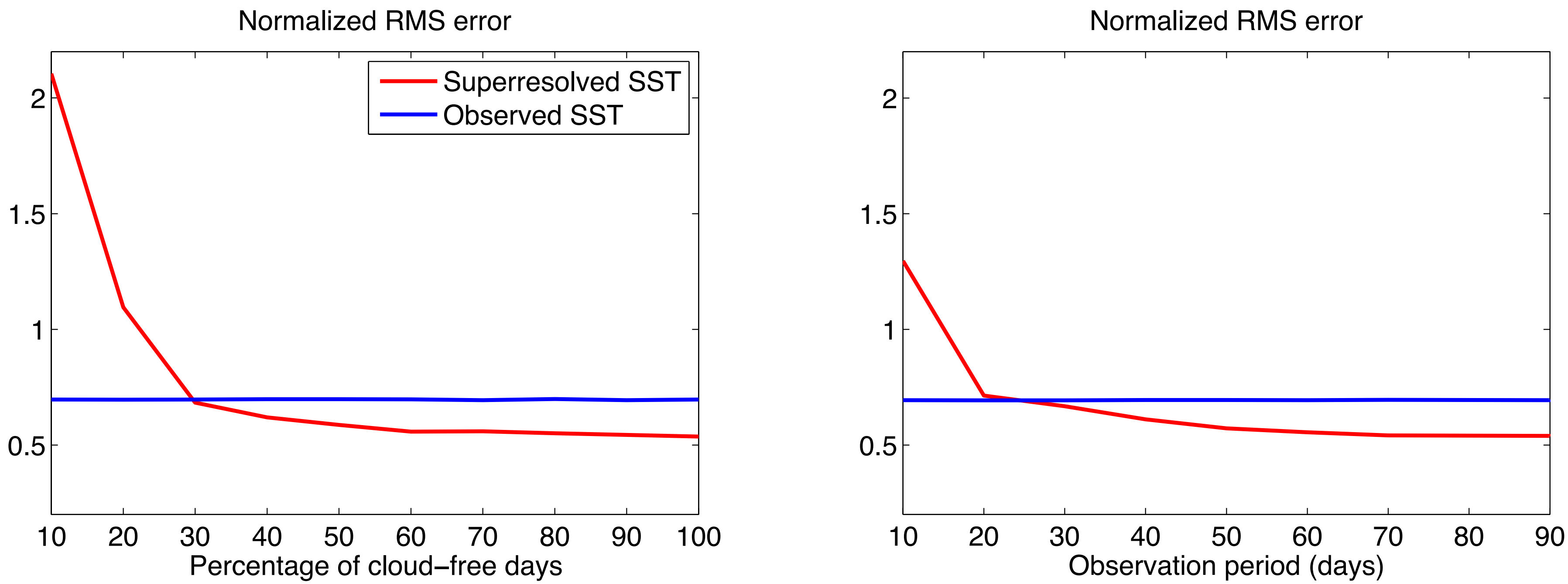
Antarctic Circumpolar Current | Gulf Stream | Equatorial Pacific



Cross-correlation: $\left\langle \left|\theta(k)-\theta^{true}(k)\right|^2 \right\rangle^{1/2} \Big/ \left\langle \left|\theta^{true}(k)\right|^2 \right\rangle^{1/2}$



Sensitivity to **clouds and observing period**:



- **Accuracy of small-scale statistics** calculated using high-resolution images depends on quality of data
- **Model effect of imperfect data** by randomly discarding frames ("clouds") or shortening observing period

Upper ocean flow reconstruction

- **SQG model:** elliptical equation for streamfunction with boundary condition given by SST

$$\nabla^2\psi + \frac{\partial}{\partial z}\left(\frac{f^2}{N^2}\frac{\partial\psi}{\partial z}\right) = 0, \qquad \left.\frac{\partial\psi}{\partial z}\right|_{z=0} = \frac{g\alpha}{f}\theta_{surf}$$

Zero interior potential vorticity anomaly

Dynamics driven by surface temperature (density)

- Lapeyre and Klein (2006), Isern-Fontanet et al. (2006): effect of interior PV anomalies in upper ocean can be modeled by replacing *N(z)/f* with an ``**effective Prandtl ratio**'' σ_0.

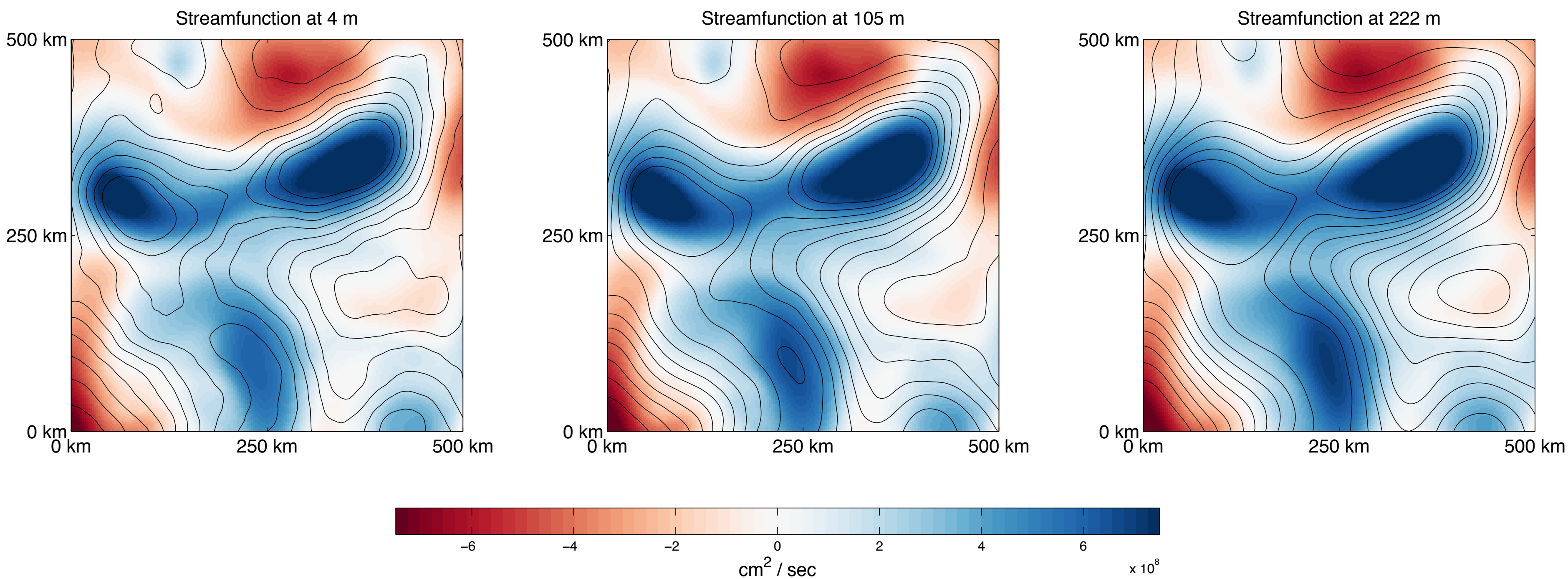
$$\hat{\psi}(k,z) = \hat{\psi}(k,0)\mathrm{e}^{\sigma_0 K z}, \qquad \hat{\psi}(k,0) = \frac{g\alpha}{f\sigma_0 K}\hat{\theta}_{surf}(k)$$

Upper ocean flow reconstruction

- Fit σ_0 by **matching EKE at surface** with low-resolution altimetry.

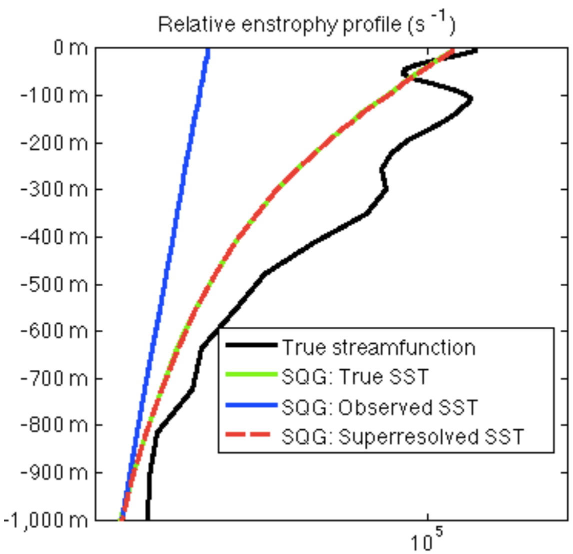
$$EKE = \frac{1}{2}\sum_{k,l} K^2 \left|\hat{\psi}_a\right|^2 = \frac{1}{2}\frac{g^2\alpha^2}{f^2\sigma_0^2}\sum_{k,l}\left|\hat{\theta}_{surf}\right|^2$$

- Geostrophic streamfunction at depth: Gulf Stream

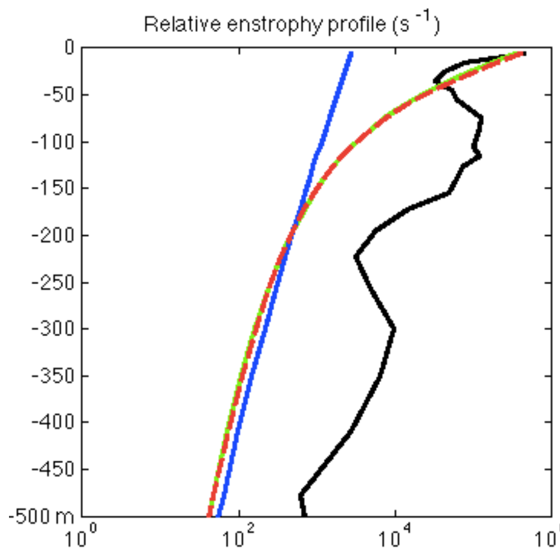


Upper ocean flow reconstruction

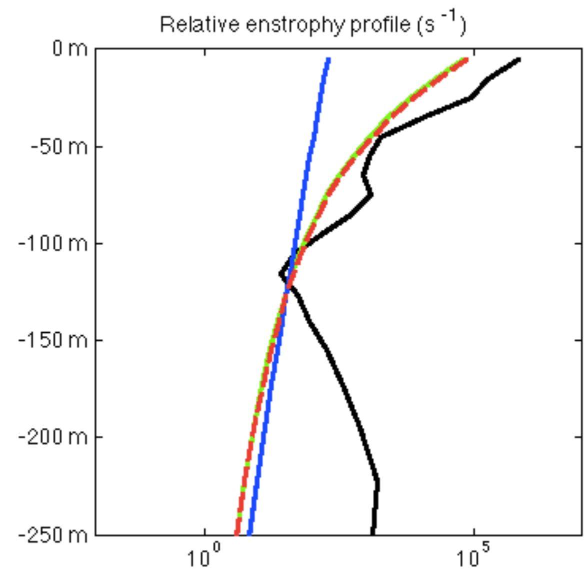
Antarctic Circumpolar Current



Gulf Stream



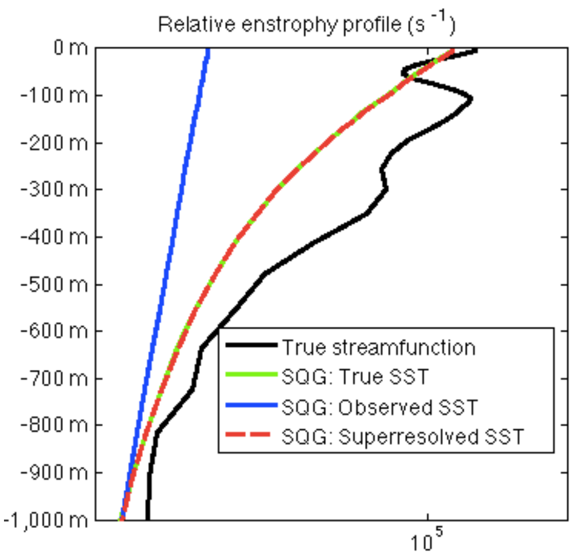
Equatorial Pacific



- Even with **perfect observations of SST,** SQG methods have a **depth of validity varies regionally**.
- Argues for **inclusion of interior dynamics**: Lapeyre (2009), Ponte and Klein (2013), Wang et al. (2013).
- However, super-resolved SST results in significantly improved **surface mode reconstruction** compared with raw observations.

Conclusions

Antarctic Circumpolar Current



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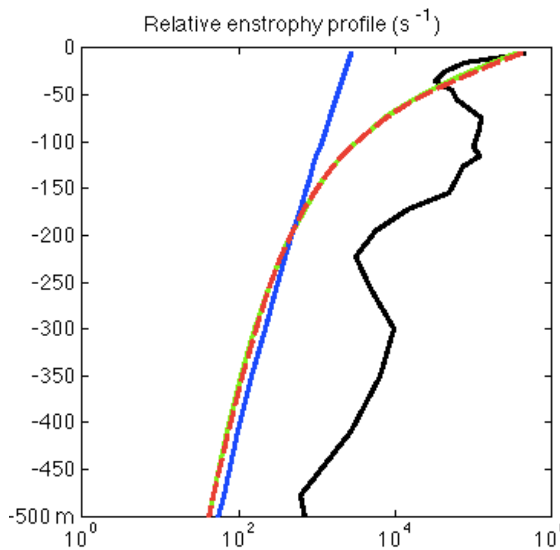

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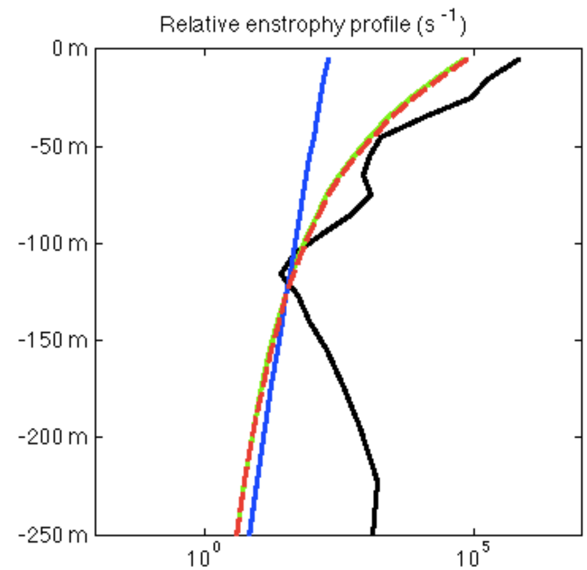

- **SQG projections** require high-resolution SST observations to resolve O(10) km ocean flow.
- Combine microwave images with statistical information from infrared observations to construct **super-resolved SST images**.
- Strong regional variation due to influence of internal dynamics. However, **``surface mode''** is well captured in all cases.